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Space–Time in a Simple Model of a Physical System

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A model of space–time is worked out starting from the two primitive concepts of preparticle and of membership relation of set theory. We obtain as derivative concepts those of space–time and inertial reference frame, also those of energy, frequency, momentum, and wavelength of a physical system in a given reference frame. Proportionality relations between energy and frequency, and between momentum and (wavelength)$^{-1}$ are shown to be satisfied in our model. The same constant of proportionality intervenes in these two relations, and we interpret it as the Planck constant expressed in a particular system of units. Energy and momentum are conserved in the usual sense, provided we consider sufficiently large regions of the space–time diagram associated to the reference frame under consideration. Lorentz transformations and Heisenberg's inequalities are discussed within the framework of our model.

1. INTRODUCTION

Few concepts in physics have proved to be more fruitful, descriptive, and unifying than space–time. This explains why there is a growing interest in foundation theories of this concept. There exist several such theories, among which are the following: Basri's deductive theory of space and time (Basri, 1966), Finkelstein's space–time code theory (Finkelstein, 1969, 1972a, 1972b, 1974; Finkelstein, Frye, and Susskind, 1974), and Penrose's theory of space–time (Penrose, 1967, 1968, 1975). Also, foundational theories of the concepts of space (Penrose, 1971; Bunge and García-Máynez, 1976) and time (Noll, 1967; Bunge 1967, pp. 93–100; García-Sucre, 1975) have been proposed. Let us give a very brief account of these theories. However, the interested reader is urged to study the above-mentioned papers together with the more general works of Russell (1969a, 1969b), Reichenbach (1957), Gonseth (1964), Bunge (1967), and

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the collection of papers in the book *The Nature of Time*, edited by Gold (1967).

Basri's theory is a relational and macroscopic theory of space–time. Starting from eight primitive concepts, this theory develops a space–time geometry. The primitive concepts of this theory are those of observer, sensation, subjective entity, objectivity, appearance event, disappearance event, coincidence of events, and signal relation. Particles, events, clocks, and length-measuring instruments are derivative concepts playing an essential role in the whole theory. General Relativity and Basri's theory appear to be compatible (Basri, 1966).

In Finkelstein's theory (Finkelstein, 1969, 1972a, 1972b, 1974; Finkelstein et al., 1974) the most basic entities are elementary processes. The way in which elementary processes form a process is represented by a network. The world is considered to be represented by a discrete complex of binary entities, which in turn represent elementary quantum processes. In this sense, the Finkelstein theory is an atomistic microscopic theory whose atoms (considered as entities that cannot be further separated in parts) are processes themselves. These basic entities of the theory are required to be both quantum and relativistic objects (Finkelstein, 1974). Minkowski's space–time and a proper time Dirac equation are obtained in a particular limit of this theory (Finkelstein et al., 1974).

The Penrose theory is also a microscopic theory of space–time (Penrose, 1967, 1968, 1975), in which the most basic entities are represented by pairs of spinor fields. Penrose has called these basic entities twistors and has proposed as one of the possible intuitive representations massless particles in free motion (Penrose, 1975). A twistor space is introduced such that every space–time point corresponds to a subset of twistors belonging to this space. In this theory, both space–time and the complex Hilbert space of quantum mechanics are derivative from the twistor space. This capacity to describe space–time and Hilbert space (two completely different kinds of spaces) from a unifying point of view is one of the main points of this theory. Another important result of Penrose's theory is that once the properties of the twistor are given, then the dimension and signature of space–time are correctly fixed (Penrose, 1975).

Theories of the relatively simpler concept of space have also been proposed. Penrose's spin network model of the physical world is an example of such theories (Penrose, 1971). According to this theory, the physical world can be seen as a network in which every junction (line) between knots (points) represents a physical entity with a well-defined angular momentum equal to $n\hbar/2$, where n is an integer.

The theory of space proposed by Bunge and García-Máynez (1976) is a foundation theory. They construct a relational theory of space from the

primitive concepts of concrete thing and of action of one thing upon another. The physical space is defined as the set of concrete things provided with a topology. This topology is defined by making use of a separation function, which in turn is defined in terms of the primitive concepts of the theory. By adding some natural assumptions, the space is constrained to have three dimensions.

The Noll theory of universal time (Noll, 1967) starts from the primitive concepts of an event and a function that pairs couples of events to real numbers. With the aid of this function, one obtains a partition of the set of events and orders the corresponding quotient set. Each equivalence class belonging to this set represents an instant. Bunge extends the theory by introducing new concepts which allow the definition of local time (Bunge, 1967, pp. 93–100, 1968).

In our model we start by assuming an atomistic type of hypothesis. We take for granted the existence of preparticles, which are considered to be the basic ingredients of any physical system. Preparticles are unchangeable physical objects without internal structure (García-Sucre, 1975, 1978a). Furthermore, preparticles do not interact with each other. The concept of interaction between systems of particles is in our model a derivative concept not defined for preparticles (García-Sucre, 1978a).

Particles are formed by preparticles in a sense to be precisely stated below. The advantages of this distinction between preparticles and particles in connection with the problem of the atomistic hypothesis in physics have been discussed in a previous paper (García-Sucre, 1978a).

Let $B \equiv \{\alpha_i | i \in I\}$, where I is a finite set of labels, be the set whose numbers α_i are all the preparticles. In previous papers the set B was assumed as a denumerably infinite set (García-Sucre, 1975, 1978a, b). However, in the present paper we take B to be a finite set, though with very many members. We have been inclined in favor of this last choice because in this way our model accounts for a general feature that real physical systems have (see Section 3). On the other hand, all the properties of our model except those related to the number of all possible evolving and nonevolving particles, which have been explained in previous papers (García-Sucre, 1975, 1978a) remain unchanged when the constraint of finiteness of B is introduced.

Note that set B has no structure. The only property that characterizes this set is that its members are just all preparticles. However, the power set $P(B)$ has a structure and is simply related to B. Since we have assumed the existence of B, then the existence of $P(B)$ follows from postulate of usual set theory. In this concern, let us say that throughout the present paper we make use of set theory in the version of Fraenkel and Bar-Hillel (Fraenkel, 1961a; Fraenkel and Bar-Hillel, 1958).

We represent a particle as a subset of $P(B)-\phi$, *where* ϕ *denotes the empty set, and a physical system as a set of such sets* (García-Sucre, 1975).

Then the set

$$p_i = \{a^i(x) | x \in X \quad \text{and} \quad a^i(x) \in P(B)\} \tag{1.1}$$

where X is a finite set of labels, represents a particle. If the set p_i can be completely ordered according to the proper inclusion relation $\subset$ between its members, we say that p_i represents an *evolving particle* (García-Sucre, 1975). In other words, an evolving particle is a *chain* (the concept of a chain is clearly discussed in Fraenkel and Bar-Hillel, 1958, pp. 128–131). On the other hand, the subsets of $P(B)$ that are not chains represent *nonevolving particles* (García-Sucre, 1975, 1978a).

We call α *state of* p_i any set $s^i(x)$ of preparticles fulfilling the relation

$$s^i(x) = a_i(x) - Ua^i(x'), \qquad x' \in X'(x) \tag{1.2}$$

where $a^i(x)$, $a^i(x') \in p_i$, and all the $a^i(x')$ with $x' \in X'(x)$ are members of p_i such that $a^i(x) \not\subseteq a^i(x')$ (García-Sucre, 1975). The α states of a particle can be ordered according to the following rule: Given the α states $s^i(x)$ and $s^i(y)$ of p_i we say that $s^i(x)$ precedes $s^i(y)[s^i(x) \prec s^i(y)]$ if $a^i(x) \subset a^i(y)$, where $a^i(x)$, $a^i(y) \in p_i$.

We denote as $\Sigma(p)$ the set of α states of a particle p. According to the above ordering rule for α states it follows that if p is an evolving particle, then $\Sigma(p)$ is a completely ordered set. However, if p is a nonevolving one then both p and $\Sigma(p)$ are partly ordered sets.

An intuitive representation of an evolving particle could be a set of closed surfaces, each one containing a finite number of preparticles. Each such closed surface represents an α state of the particle under consideration. The preparticles inside a closed surface are the preparticles belonging to the α state represented by such a closed surface. The closed surfaces themselves stand only to point out the preparticles belonging to the α state represented by them. Roughly speaking we can say then that the closed surfaces are only used to represent the fact that preparticles inside them form sets, just those sets of preparticles which are the α states of the particle under consideration. Furthermore, all the closed surfaces representing the α states of a given particle are linked in sequence by a flexible arrow, which specifies the ordering of these α states. Recall that the order in which the α states of an evolving particle p appear is completely determined, once the particle p is given, by the way in which the members $a(x)$ of p are properly included in each other. Therefore, the arrows linking closed surfaces serve *only to give a graphic representation* of the order in which α states appear for a particle.

Following the same conventions, a set of particles will appear as a collection of arrows passing over closed surfaces. If closed surfaces related to different particles overlap (because there exist preparticles belonging to both closed surfaces) then these particles are connected. Since we have said that an arbitrary set of particles represents a physical system, the structure of a physical system is characterized by both the ordering of the α states of its particles and the way in which these particles are connected to each other. Note that according to equation (1.2), which defines α states of a particle p_i, it can be seen that for an evolving particle p_j the elements $a^j(y)$ of p_j will be represented within the above conventions as unions of closed surfaces (see Theorem 1 demonstrated in Section 2). On the other hand, each nonevolving particle will appear in the above representation as either a collection of arrows linking closed surfaces or a collection of closed surfaces not linked by arrows, or as a mixture of the two cases. This follows from the fact that nonevolving particles and the corresponding sets of α states are represented by sets that are only partly ordered. In other words, within the above conventions an evolving particle appears as a graph with only one branch and a nonevolving particle appears as either a ramified graph, or several one-branch graphs, or closed surfaces without connecting arrows, or mixtures of these three cases.

According to our definition of a set representing a system of particles, two main cases may arise. Given a set $B' \subseteq B$ of preparticles, a set S whose elements are subsets of $P(B')$—and thus representing a system of particles —may be such that any $p \subseteq P(B')$ is an element of S. In this case, we say that S is *complete* with respect to the set of preparticles B'. The complementary case also occurs in our model and, in fact, given a set of preparticles $B' \subseteq B$, only one of the sets S that can be constructed using as elements subsets of $P(B')$ is complete. All other such sets representing physical systems are not complete. Note that if a set S is complete with respect to a set B', then this set is a *lattice* since $\phi \in S$ and both the union or the intersection of any two elements of S again yield elements of S (Birkhoff, 1961).

We will see in the next section that according to the model outlined above the concepts of field produced by a physical system (García-Sucre, 1978a), of space–time, and of inertial reference frame, can be obtained as derivative concepts starting from the primitive concepts of preparticle and of membership relation $\in$ of the set theory. Here we also prove a property of evolving particles and discuss the concepts of field, space–time, and reference frame. In Section 3 we analyze the concepts of energy and frequency, momentum and wavelength of a physical system in a given frame. Also, we briefly analyze the problem of the pointlike aspect that particles sometimes present, in contrast with the extended character that they manifest at some other times.

Finally, let us underline that our model of space–time is relational in the sense that the disappearance of preparticles would likewise entail the disappearance of particles, physical systems, fields, and space–time. In other words, space–time does not have an existence independent from that of the most elementary components of matter.

2. FIELD AND SPACE–TIME

Let us start this section by proving a property of evolving particles.

Theorem 1. For any evolving particle p_i with an ordered set of α states $\Sigma(p_i)=(s^i(x),s^i(y),\ldots,s^i(v))$ one has $p_i=(s^i(x),s^i(x)\cup s^i(y),\ldots,s^i(x)\cup s^i(y)\cup\cdots\cup s^i(v))$. Conversely, given a partition $\pi=\{t_i|i\in I,t_i\subseteq B'$ and $t_i\cap t_j=\varnothing$ for any $i,j\in I\}$ of a set $B'\subseteq B$, where I is a finite set of integers, then a set $c\equiv(t_k,t_k\cup t_l,\ldots,t_k\cup t_l\cup\cdots\cup t_s|k,l,\ldots,s\in I)$ is an evolving particle with a set of states given by $(t_k,t_l,\ldots,t_s)$.

Proof. There exists a one-to-one mapping between the sets $\Sigma(p_i)$ and p_i, such that the members of both sets are in the correspondence $s^i(x)\leftrightarrow a^i(x)$ established by equation (1.2)

$$s^i(x)=a^i(x)-\cup a^i(x'),\qquad x'\in X'(x) \tag{2.1}$$

where $s^i(x)\in\Sigma(p_i),a^i(x)\in p_i,a^i(x)\not\supseteq a^i(x')$. To see this let us first recall Theorem 1 of a previous paper (García-Sucre, 1978a) according to which given two different elements $a^i(x)$ and $a^i(y)$ of an evolving particle p_i, then the α states $s^i(x)$ and $s^i(y)$ related, respectively, to $a^i(x)$ and $a^i(y)$ by equation (2.1) are disjoint sets, i.e., $s^i(x)\cap s^i(y)=\varnothing$. Therefore, $a^i(x)\neq a^i(y)$ implies $s^i(x)\neq s^i(y)$.

Conversely, $s^i(x)\neq s^i(y)$ implies that $a^i(x)\neq a^i(y)$. This is equivalent to proving that from $a^i(x)=a^i(y)$ it follows that $s^i(x)=s^i(y)$. To see this last implication let $T(a^i(x))\equiv\{a^i(x')|x'\in X'(x)\}$ and $T(a^i(y))\equiv\{a^i(y')|y'\in X'(y)\}$ be the sets to which belong all the members of p_i fulfilling, respectively, that $a^i(x')\not\supseteq a^i(x)$ and $a^i(y')\not\supseteq a^i(y)$ [see the definition of α state, equation (2.1)]. Then, from $a^i(x)=a^i(y)$ it follows that $T(a^i(x))=T(a^i(y))$ since the members of both sets fulfill the same property $\mathcal{P}$ determining each set (Fraenkel, 1961b). Therefore,

$$\bigcup_{x'\in X'(x)} a^i(x')=\bigcup_{y'\in X'(y)} a^i(y')$$

which together with $a^i(x)=a^i(y)$ and equation (2.1) leads to $s^i(x)=s^i(y)$.

Thus we have that there exists a one-to-one mapping between $\Sigma(p_i)$ and p_i. Furthermore, the particular mapping between $\Sigma(p_i)$ and p_i with which we have been concerned above establishes a similarity relation between $\Sigma(p_i)$ and p_i [let us recall that a similarity relation between sets is a binary relation between sets such that there exists a one-to-one mapping between them that preserves the order in which the members appear in each of these sets (Fraenkel, 1961, pp. 134–138)].

Since $\Sigma(p_i)$ has a first member, say $s^i(x)$, and there exists a similarity relation between $\Sigma(p_i)$ and p_i, then p_i must also have a first member which according to equation (2.1) is $a^i(x)$. Furthermore, $s^i(x)=a^i(x)$ since p_i is a chain, $a^i(x)$ is its first member, and therefore any other member $a^i(z)$ of p_i is such that $a^i(x)\subset a^i(z)$. Then $\cup_{x'\in X'(x)}a^i(x')=\varnothing$ in equation (2.1) and thus $s^i(x)=a^i(x)$.

We have in this way proven the first part of the theorem for the first member of p_i. Let us proceed by induction and assume that the property is satisfied by the nth member of p_i, i.e.,

$$a^i(r)=s^i(x)\cup s^i(y)\cup \overset{n}{\cdots} \cup s^i(r) \tag{2.2}$$

The relation between the $(n+1)$th α state $s^i(t)$ and $(n+1)$th member $a^i(t)$ of p_i is given by equation (2.1):

$$s^i(t)=a^i(t)-\bigcup_{t'\in X'(t)} a^i(t') \tag{2.3}$$

From the fact that p_i is a chain and that $a^i(r)$ immediately precedes $a^i(t)$ in p_i, one obtains $\cup_{t'\in X'(t)}a^i(t')=a^i(r)$. Then equation (2.3) can be written as $s^i(t)=a^i(t)-a^i(r)$, and since $a^i(r)\subset a^i(t)$ we get $a^i(t)=a^i(r)\cup s^i(t)$. This relation and equation (2.2) entail

$$a^i(t)=s^i(x)\cup s^i(y)\cup\cdots\cup s^i(t) \tag{2.4}$$

which completes the proof of the first part of the theorem.

Reciprocally, the set $c=\{t_k, t_k\cup t_l,\ldots,t_k\cup t_l\cup\cdots\cup t_s\}$, where $t_k,t_l,\ldots,t_s$ are members of a partition π of a set $B'\subseteq B$, represents a particle since the members of c are subsets of B, and thus c is a subset of the power set $P(B)$. Also, the set c represents an evolving particle since its members can be completely ordered by the proper inclusion relation $\subset$. Finally, from equation (2.1), and on account of the ordering rule $s^i(x)\prec s^i(y)$ if and only if $a^i(x)\subset a^i(y)$, one obtains, following similar steps as above, that $\Sigma(c)=(t_k,t_l,\ldots,t_s)$, which completes the proof.

> *Corollary 1.* Any evolving particle p_i is univocally determined by its set of α states $\Sigma(p_i)$.

Proof. First, two different evolving particles p_i and p_j cannot have the same set of α states, since in that case $\Sigma(p_i)=\Sigma(p_j)$ and according to the first part of Theorem 1 we should have $p_i=p_j$. Conversely, an evolving particle p_i cannot have more than one set of α states. To see this, assume that an evolving particle can have more than one set of α states. Let us choose two of these sets, which can be written as

$$\Sigma(p_i)=(s^i(x_1),s^i(x_2),\ldots,s^i(x_n)) \tag{2.5}$$

and

$$\Sigma'(p_i)=(s^i(y_1),s^i(y_2),\ldots,s^i(y_m)) \tag{2.6}$$

Then, from Theorem 1 and equations (2.5) and (2.6) we can write, respectively,

$$p_i=(s^i(x_1),s^i(x_1)\cup s^i(x_2),\ldots,s^i(x_1)\cup s^i(x_2)\cup\cdots\cup s^i(x_n)) \tag{2.7}$$

and

$$p_i=(s^i(y_1),s^i(y_1)\cup s^i(y_2),\ldots,s^i(y_1)\cup s^i(y_2)\cup\cdots\cup s^i(y_m)) \tag{2.8}$$

Since these two ordered sets are equal we must have that $n=m$ and

$$s^i(x_1)=s^i(y_1)$$

$$s^i(x_1)\cup s^i(x_2)=s^i(y_1)\cup s^i(y_2)$$

$$\vdots$$

$$s^i(x_1)\cup s^i(x_2)\cup\cdots\cup s^i(x_n)=s^i(y_1)\cup s^i(y_2)\cup\cdots\cup s^i(y_n)$$

Recall now that the α states of an evolving particle are disjoint sets (see Theorem 1 of a previous work, García-Sucre, 1978a). Then, from the above equalities it follows that $s^i(x_1)=s^i(y_1), s^i(x_2)=s^i(y_2),\ldots,s^i(x_n)=s^i(y_n)$, and therefore $\Sigma(p_i)=\Sigma'(p_i)$.

In order to describe the structure of a system S, we use the concept of *point of crossing in a physical system*. [In a previous paper (García-Sucre, 1978a) we have called it complex, yet we prefer the present denomination because it suggests an appropriate intuitive representation.] A point of crossing in a physical system is represented by an ordered pair $(s^i(x);\pi_x^i(S))$, where $s^i(x)$ is an α state of a particle belonging to S, and $\pi_x^i(S)$ is the subset of S such that if $p\in\pi_x^i(S)$, then there exists at least one $s\in\Sigma(p)$ fulfilling $s\cap s^i(x)\neq\varnothing$. We call $s^i(x)$ the center, and $\pi_x^i(S)$ the π set of the point of crossing $(s^i(x);\pi_x^i(S))$, which we also denote as $\sigma(s^i(x);S)$.

Two points of crossing $\sigma(s^i(x); S)$ and $\sigma(s^j(y); S)$ are *connected* with each other if there exists at least one particle $p \in S$ such that either there exists a $s \in \Sigma(p)$ for which we have $s \cap s^i(x) \neq \emptyset$ and $s \cap s^j(y) \neq \emptyset$, or there exists at least two $s', s'' \in \Sigma(p)$ such that $s' \cap s^i(x) \neq \emptyset$ and $s'' \cap s^j(y) \neq \emptyset$.

The *structure-set* $\Sigma\Sigma(S)$ of a system S is the set whose members are all the points of crossing in S (García-Sucre, 1978a).

Two particles p and p' are *similar or have the same structure* if there exists a one-to-one mapping ψ between $\Sigma(p)$ and $\Sigma(p')$ such that ψ both preserves the order in each of these sets and puts into correspondence α states with the same number of preparticles. We call ψ a similarity mapping.

Let $\sigma = (s^i(x); \pi_x^i(S))$ and $\sigma' = (s^j(y); \pi_y^j(S))$ be two points of crossing in S presenting identical structures. More precisely, we say that σ and σ' are *similar or have the same structure*—which we denote as $\sigma \frown \sigma'$—if there exists a one-to-one mapping φ between $\pi_x^i(S)$ and $\pi_y^j(S)$ fulfilling the conditions (i) if $p_i \overset{\varphi}{\leftrightarrow} p_j$, then p_i and p_j are similar particles; (ii) there exists a similarity mapping $\Sigma(p_i) \overset{\psi}{\leftrightarrow} (p_j)$ which puts into correspondence a $s \in \Sigma(p_i)$ with a $s' \in \Sigma(p_j)$ and $s \cap s^i(x) \neq \emptyset$ and $s' \cap s^j(y) \neq \emptyset$.

In a previous paper we have required a further condition for $(s^i(x); \pi_x^i(S)) \sim (s^j(y); \pi_y^j(S))$, which refers to other crossings of particles than those occurring over the centers of these two points of crossing (García-Sucre, 1978a). Here we have omitted this kind of condition. Instead we have preferred that for each point of crossing in S only its center becomes relevant in connection with crossings of particles. Therefore, the structure of a point of crossing is determined by the particles crossing over its center.

In a previous paper we have given arguments in favor of representing by $\Sigma\Sigma(S)/\sim$ the field produced by a system S (García-Sucre, 1978a). Recall that $\sim$ stands for the relation of similarity between points of crossing.

In both sets $\Sigma\Sigma(S)/\sim$ and $\Sigma\Sigma(S)$ the structural features of S are apparent. The elements of $\Sigma\Sigma(S)$ are points of crossing and the set $\Sigma\Sigma(S)$ can be visualized as a network whose knots are the centers of the points of crossing in S. The junctions between these knots stand for the particles of S connecting points of crossing in S. On the other hand, the elements of $\Sigma\Sigma(S)/\sim$ are equivalence classes of points of crossing. Here again we can visualize $\Sigma\Sigma(S)/\sim$ as a network, each knot of which represents the union of the centers of the points of crossing belonging to the same element of $\Sigma\Sigma(S)/\sim$. The junctions in such a network stand for the particles of S connecting points of crossing belonging to different equivalence classes. In fact, we consider each equivalence class belonging to $f = \Sigma\Sigma(S)/\sim$ as a point of the field represented by f; the connections between two points

$x, x' \in f$ being given by the particles of S connecting points of crossing belonging, respectively, to x and x' (García-Sucre, 1978a).

Furthermore, each one of the sets $\Sigma\Sigma(S)/\sim$ and $\Sigma\Sigma(S)$ have the following positive features in order to represent the field produced by a system S (García-Sucre, 1978a): (i) they are both such that their elements, points of crossing in the case of $\Sigma\Sigma(S)$ and equivalence classes of points of crossing in the case of $\Sigma\Sigma(S)/\sim$, are constructed out of particles belonging to S. This establishes a direct relation between physical systems and the fields they produce; (ii) a topology can be ascribed to each of the sets $\Sigma\Sigma(S)/\sim$ and $\Sigma\Sigma(S)$ once S is given. The topology of either $\Sigma\Sigma(S)/\sim$ or $\Sigma\Sigma(S)$ will be determined by the way in which the elements of $\Sigma\Sigma(S)/\sim$ or $\Sigma\Sigma(S)$ are, respectively, connected; and, (iii) the concept of interaction between two physical system S and S' can be obtained as a clear-cut derivative concept using either one of the two possibilities of representing fields.

If we represent by $f = \Sigma\Sigma(S)/\sim$ and $f' = \Sigma\Sigma(S')/\sim$ the fields produced by S and S', the definition of interaction between S and S' is as follows (García-Sucre, 1978a): the systems S and S' interact if and only if $f \cup f' \neq \Sigma\Sigma(S \cup S')/\sim$. Equivalently, we say in this case that f and f' are coupled to each other. In the complementary case, i.e., if $f \cup f = \Sigma\Sigma(S \cup S')/\sim$, we say that S and S' do not interact or equivalently that f and f' are uncoupled.

In the case that we choose $\Sigma\Sigma(S)$ instead of $\Sigma\Sigma(S)/\sim$ to represent the field produced by S, the interaction between two physical systems can be defined along the same lines as in the case of $\Sigma\Sigma(S)/\sim$.

Notwithstanding the features that $\Sigma\Sigma(S)/\sim$ and $\Sigma\Sigma(S)$ have in common we have been inclined in favor of $\Sigma\Sigma(S)/\sim$ in order to represent the field produced by S (García-Sucre, 1978a). We explain below the main reasons for this choice.

First, recall that to an element x of $\Sigma\Sigma(S)/\sim$ is associated the structure that is shared by all the points of crossing belonging to the equivalence class x. Since $\Sigma\Sigma(S)/\sim$ is necessarily a partition of $\Sigma\Sigma(S)$ there cannot occur overlapping between different elements of $\Sigma\Sigma(S)/\sim$. Each element of $\Sigma\Sigma(S)/\sim$ can be labeled by its own structure without incurring ambiguity, since the structure of a given point of $\Sigma\Sigma(S)/\sim$ is necessarily different from the structure of any other point of $\Sigma\Sigma(S)/\sim$. This is closely related to the point of view that one adopts in general relativity, according to which the labeling of the space–time points proceeds by taking into account what happens in their surroundings (Misner, Thorne, and Wheeler, 1970). The structure of a point x of $\Sigma\Sigma(S)/\sim$ is determined by the structure of the particles of S crossing over centers of points of crossing belonging to x (in short, we will say: the particles of S crossing over x). In the representation of $\Sigma\Sigma(S)/\sim$ as a network that we

have mentioned above it is clear that the particles of S crossing over x can be interpreted as being part of the surrounding of the network knot corresponding to x. However, we have left out the neighboring points of x in order to characterize x, which would be more approximate to the method of labeling the space–time points in general relativity. This is because in our model it is sufficient to consider only the structure of the particles crossing over a point of a field to characterize this point without ambiguity with respect to all other points of the same field.

On the other hand, if we represent the field produced by S with the set $\Sigma\Sigma(S)$ instead of $\Sigma\Sigma(S)/\sim$, the only way we have for labeling two points of crossing having the same structure without introducing extra elements to $\Sigma\Sigma(S)$—such as marks, letters, numbers, etc.—is to use the preparticles entering in each such points of crossing. This can be done in principle since preparticles have been assumed to be elements of the same set B, and therefore preparticles must be distinct from each other by set definition. However, this way of labeling would be in conflict with our definition of interaction between systems of particles. We have said that the concept of interaction in our model is not defined for preparticles. There is no meaning in saying that two preparticles interact, or that a preparticle interacts with a system of particles, etc. In our model only systems of particles can interact with each other, and we also say that two particles can interact in the sense that two systems each having a unique particle as element can interact with each other. One is then inclined to conclude that preparticles cannot be detected since it seems difficult that the detection of a physical entity can occur without interacting with this entity (we define below what we understand precisely by detection of a particle when we define detectors of particles as a particular case of physical systems). If we admit that there is no way to detect preparticles in the framework of our model this should be a good reason not to characterize either point of crossing belonging to $\Sigma\Sigma(S)$ or points of $\Sigma\Sigma(S)/\sim$ by the preparticles entering in them.

Note that if we represent a field produced by S with the set $\Sigma\Sigma(S)/\sim$ we are implicitly assuming that when the field under consideration has more than one point it is precisely because there exist points of crossing in S that differ in structure. Therefore, without introducing any extra elements to a field one has that each of its points is completely characterized with respect to the remaining points of the same field. We then introduce the following postulate:

Postulate 1. Any region of the physical world is formed by field points and two such regions are distinct from each other only because the field points entering in one of them have different structures from those of the field points entering in the other region.

Let p be a particle that does not belong to S. We can then ask: how does system $S' = \{p\}$ affect the field $f = \Sigma\Sigma(S)/\sim$ due to the interaction between S and S'? Following our definition of interaction between fields, consider the field $f' = \Sigma\Sigma(S \cup S')/\sim$ which may differ from $f = \Sigma\Sigma(S)/\sim$ in several ways:

(i) When going from S to $S \cup \{p\}$ the structure of those points of crossing in S over which p passes may change.

(ii) A point of crossing belonging to an equivalence class of $\Sigma\Sigma(S)/\sim$ having a given structure, when modified by p may become an element of an equivalence class of $\Sigma\Sigma(S \cup S')/\sim$ having a different structure.

(iii) The number of points of $f = \Sigma\Sigma(S)/\sim$ may be different from the number of points of $f' = \Sigma\Sigma(S \cup S')/\sim$. It may occur that the number of points of f' is either smaller or larger than those of f. The first case arises when going from S to $S \cup \{p\}$ the particle p destroys the dissimilarity between some points of crossing in S in such way that some equivalence classes of $\Sigma\Sigma(S)/\sim$ become empty sets without the appearance of new equivalence classes when one considers the set $\Sigma\Sigma(S \cup S')/\sim$. The second case, in which f' has more points than f, occurs when going from S to $S \cup \{p\}$ the particles p destroy the similarity between points of crossing in S giving rise to points of crossing having structures different from those of any point of crossing of S.

On the other hand, if we represent fields by sets $\Sigma\Sigma(S)$, even if (i) applies again, both (ii) and the case in which f' has a smaller number of points than f mentioned in (iii) do not apply to this case. It can be easily seen that $\Sigma\Sigma(S \cup \{p\})$ can only have either the same number or more points of crossing than $\Sigma\Sigma(S)$ since there are only two possibilities: either any α state of p is a center of a point of crossing in S or there exist α states of p that are not such centers. These two cases correspond to $\Sigma\Sigma(S \cup \{p\})$ having, respectively, an equal or larger number of points of crossing than $\Sigma\Sigma(S)$.

The above discussion also applies to the case in which $f = \Sigma\Sigma(S)/\sim$ interacts with a system of many particles instead of only one particle as discussed above. We can then say that in choosing $\Sigma\Sigma(S)/\sim$ instead of $\Sigma\Sigma(S)$ to represent the field produced by S we are selecting an alternative that proves richer in relation to the ways in which a field can be modified by the interaction with another system of particles. And we will see in Section 3 that the point (ii) above is important in relation to the process of detection of particles in a given reference frame. Also, it is relevant to the fact that in our model physical systems, in a detection process, sometimes behave as if they were pointlike objects and at other times they appear as spatial extended entities. Examining this question in detail would lead us to the problem of whether quantum mechanics, or at least some essential

traits of this theory, finds a natural place in our model. This we will try to answer in a forthcoming paper (García-Sucre, 1979a). Here we limit ourselves to a discussion of the concept of detector of physical systems and to a description of the way in which physical systems appear in a detection process. When the detected physical system has a unique particle as element we also speak of detection of a particle. We also discuss in Section 3 a trait of particles in our model that allows us to ascribe both a frequency and a wavelength to every particle and more generally to every physical system in a reference frame.

The *intensity of a field at one of its points*, say x, is given by the number of preparticles belonging to the union of the centers of all the points of crossing belonging to this point x. We denote the intensity of a field f at this point as $I(x;f)$.

A *field f is said to be homogeneous* if the intensity of f has the same value at every point of f, i.e., if $I(x;f)=I(y;f)$ for every pair of points $x,y \in f$. When the complementary case stands we say that f is *nonhomogeneous*.

When a particle p has at least an α state which yields a nonempty intersection with the center of a point of crossing belonging to a point x of a field f, we have said that p crosses over the point x of f. Let $x_1,\ldots,x_i,\ldots,x_n$ be the points of f such that p crosses over them. The trajectory of p in f can be defined as the set of these points ordered according to the order in which the α states of p appear in the set $\Sigma(p)$ (García-Sucre, 1978a). It is clear that the trajectory of p in f will be a completely ordered set if (a) each α state of the particle p gives rise to either only one or none of the points of the trajectory of p in f; and (b) all such α states are ordered in $\Sigma(p)$. Thus, we can also have trajectories that are only partly ordered sets. We denote as T_p^f the set representing the trajectory of a particle p in the field f.

Given a collection of fields $f_1,\ldots,f_i$, respectively, produced by the system $S_1,\ldots,S_i$, we call a *space–time associated to a collection of fields* $f_1,\ldots,f_i$ to the field produced by the physical system $S=S_1\cup\cdots\cup S_i$. We denote such a space–time as $ST(S)$.

According to the above defintion, the space–time $ST(S)$ is the global field with respect to the collection $f_1,\ldots,f_i$ associated to $ST(S)$, since the fields $f_1,\ldots,f_i$ are, respectively, produced by the physical systems $S_1,\ldots,S_i$, and one has that $S=S_1\cup\cdots\cup S_i$. Also note that $ST(S)$ depends upon the physical system S and therefore upon the collection of physical systems producing the fields $f_1,\ldots,f_i$. To be sure, we can also speak of the space–time produced by the physical system resulting from the union of all physical systems. This particular case of our definition will be closer to the usual way of conceiving space–time as the unique physical entity in which

the entire universe spreads itself. Our definition opens the possibility of considering also space–times as "large" as necessary according to the fields relevant to the problem at hand.

Since according to our definition a space–time is a global field with respect to a given collection of fields, all that we have discussed above in connection with fields applies equally to space–time. In particular, let us underline that the elements of a space–time are constructed from particles only, that every point of a space–time is completely characterized with respect to any other point of the same space–time by difference between the structure of these points, and that a space–time has a topology characterized by the way in which its points are connected to each other. On the other hand, we have not assumed that space–time is uniform. This will depend in our model on the systems $S_1,\ldots,S_i$ producing the fields $f_1,\ldots,f_i$ with respect to which the global field $ST(S)=\Sigma\Sigma(S)/\sim$, where $S=S_1\cup\cdots\cup S_i$, is defined. Depending on S, the space–time $ST(S)$ may be found anywhere between the two following exteme cases: either $ST(S)$ presents all kind of irregularities such as cuts and strong nonhomogeneities or $ST(S)$ is uniform in the sense that it does not present any cuts or strong nonhomogeneities. Therefore, in our view as far as we have particles we also have space–time no matter the extent to which it may be fragmentary. In this sense, according to our definition of field and space–time it can be easily seen that if to S belong very many particles giving rise to a large number of points of crossing with different structures, then the space–time $ST(S)$ may be expected to approach uniformity and to have a large extension. If, on the other hand, the system S under consideration is "small" it may occur that $\Sigma\Sigma(S)/\sim$ has few points poorly connected to each other.

Since a space–time is a global field we will also speak of homogeneity and nonhomogeneity of a space–time. Also, we will distinguish between homogeneity and connectedness of a space–time. We say that *a space–time is connected* if it does not present any cut. Note that homogeneity is related only to the intensity of a field in each of its points.

One of the main reasons to define space–time as we have done above is that according to our definition of reference frame given below, a reference frame can be seen as a physical entity that is submerged in a given space–time. In this sense, we adopt the usual point of view that a reference frame plays the role of a "window" through which we see a part of a space–time. Furthermore, our definition of reference frame will lead us to easily characterize what we understand by an inertial frame, and to relate it to the usual way in which inertial frames are defined.

Note that since a space–time is also a field we can also speak of trajectory of a particle in a space–time. Consider a collection of physical

systems $S_1, S_2, \ldots, S_m$ which are included in the system S associated to the space–time $ST(S) = \Sigma\Sigma(S)/\sim$. Let τ be the set of all evolving particles belonging to $S_1 \cup S_2 \cdots S_n$ whose trajectories in $ST(S)$ do not cross each other, i.e., such that $p_i, p_j \in \tau$ implies that $T^f_{p_i} \cap T^f_{p_j} = \phi$. On the other hand, let ε be the set of all evolving particles belonging to $S_1 \cup S_2 \cdots \cup S_n$ such that any $p' \in \varepsilon$ has a trajectory in $ST(S)$ which either intersects the trajectory in $ST(S)$ of a particle $p \in \tau$ in only one point or does not intersect it at all, i.e., for $p \in \tau$ and $p' \in \varepsilon$ one has that $T^f_p \cap T^f_{p'}$ is either empty or only one point of f belongs to it. If each α state of the particles belonging to τ yields nonempty intersections with points of crossing belonging to only one point of $ST(S)$, then the trajectories of the particles belonging to τ in f will be completely ordered sets. Note, on the other hand, that particles belonging to ε induce orderings of collections of particle trajectories belonging to τ. In order to see this, recall that the trajectory of a $p' \in \varepsilon$ intersects the trajectory of a $p \in \tau$ at just one point or not at all. For instance, let $T^f_{p'} = (x_1, \ldots, x_i, \ldots, x_m)$ be the completely ordered set representing the trajectory of p' in f. Assume that the particles $p_a, \ldots, p_g, \ldots, p_t \in \tau$ are such that $T^f_{p_a} \cap T^f_{p'} = \{x_1\}, \ldots, T^f_{p_g} \cap T^f_{p'} = \{x_i\}, \ldots, T^f_{p_t} \cap T^f_{p'} = \{x_m\}$. Then the collection of trajectories $T^f_{p_a}, \ldots, T^f_{p_g}, \ldots, T^f_{p_t}$ of particles belonging to τ can be ordered according to the following rule: $T^f_{p_g} \prec T^f_{p_t}$ if $T^f_{p_g} \cap T^f_{p'} = \{x_i\}, T^f_{p_t} \cap T^f_{p'} = \{x_m\}$ and $x_i \prec x_m$ in the ordered set $T^f_{p'}$. This yields the completely ordered set of trajectories $(T^f_{p_a}, \ldots, T^f_{p_g}, \ldots, T^f_{p_t})$.

We say that trajectories of particles belonging to ε connect trajectories of particles belonging to τ. When two such trajectories are distributed in the ordered set under consideration in such a way that one of them is either the immediate successor or the immediate antecedent of the other, then we say that these two trajectories in f are *immediately connected* to each other by a particle of ε. A given trajectory of T^f_p with $p \in \tau$ can be immediately connected to either none or one or several trajectories of particles belonging to τ. We define the number of all such trajectories as the order of immediate connection of the trajectory T^f_p with respect to τ and ε, which we denote as $\nu^{\varepsilon}_{\tau}(T^f_p)$.

A reference frame R_f in a space–time $f = ST(S)$ with respect to a collection of physical systems $S_1, S_2, \ldots, S_n$ is specified by the set $R_f \equiv (R^0_f; \tau; \varepsilon)$, where $R^0_f \equiv \cup_{p \in \tau} T^f_p \subseteq ST(S)$ and τ, $\varepsilon \subset S_1 \cup S_2 \cup \cdots \cup S_n$ are such that the following conditions hold:

(i) The trajectories T^f_p where $p \in \tau$ are completely ordered sets.
(ii) There exists a one-to-one mapping χ between any two trajectories $T^f_{p_i}$ and $T^f_{p_j}$, where $p_i, p_j \in \tau$, which preserves the order in each of them.

(iii) A point of f appears at the most in one trajectory T_p^f with $p \in \tau$.

(iv) The trajectories $T_{p'}^f$ with $p' \in \varepsilon$ are completely ordered sets and are such that given any trajectory T_p^f with $p \in \tau$, then $T_{p'}^f$ either does not cross T_p^f or crosses it at just one point.

(v) Once a collection of trajectories T_p^f with $p \in \tilde{\tau}(p') \subset \tau$ have been ordered by a particle $p' \in \varepsilon$ according to the rule given above, this ordering remains unaltered when any other particle of ε is considered. Furthermore, $\cup_{p' \in \varepsilon} \tilde{\tau}(p') = \tau$.

(vi) For any T_p^f with $p \in \tau$ one has that $\nu_\tau^\varepsilon(T_p^f)$ is equal to either $2n$ or $2n-2$, where n is an integer.

The above conditions that the set τ and ε must fulfill are very restrictive. This fact is related to the feature of our model that the same preparticle may enter in more than one point of the field f (to be specific, we say that a preparticle α_i enters in a point $x \in f$ when α_i belongs to the union of the centers of the points of crossing belonging to x). The same preparticle α_i may belong to several points of f since, first, the only condition that a point of crossing σ must fulfill in order to belong to a point $x \in f$ is to have the same structure as any other $\sigma' \in x$ and, second, the centers of points of crossing having either the same or different structures may have nonempty intersections. Therefore, the condition (i) above is a very restrictive one since a trajectory T_p^f is a completely ordered set if both p is an evolving particle and each α state of p has nonempty intersections with centers of the points of crossing belonging to only one point of f. Yet this last condition cannot be fulfilled by an α state to which a preparticle entering in more than one point of f belongs. In the more general situation where the above case occurs, reference frames can be seen as follows. To an α state s_i of an evolving particle p corresponds the cluster of those points of f in which enter preparticles belonging to the α state s_i. The clusters are ordered according to the order of the α states of p. Then we obtain a reference frame in which clusters of points play the same role as the points in the above definition. If in each of these clusters enters a very large number of field points we say that the reference frame in question corresponds to a macroscopic scale. On the other hand, if we also want to have a reference frame structure inside each cluster we can use a pair of sets $\tilde{\tau}$ and $\tilde{\varepsilon}$ for each cluster and discard any connection between field points appearing in different clusters. For the sake of simplicity, the above remark will be hereafter taken for granted and we will always refer to our definition of reference frame where conditions (i)–(vi) appear.

We call a point of $R_f \equiv (R_f^0; \tau; \varepsilon)$ any point belonging to R_f^0. The distinction between R_f and R_f^0 is necessary because a reference frame R_f is specified both by its points (i.e., by the elements belonging to R_f^0) and by

the way in which these points are connected to each other (i.e., by τ and ε). Note that a point of R_f belongs not to R_f but to R_f^0.

In relation to the condition (vi) above we say that a reference frame R_f in a space–time $f = ST(S) = \Sigma\Sigma(S)/\sim$ is an n-dimensional reference frame with an $(n-1)$-dimensional boundary if for some trajectories $T^f_{p_i}$, where $p_i \in \tau_a \subset \tau$, one has that $\nu^{\varepsilon}_{\tau}(T^f_{p_i}) = 2n$, and for some other trajectories $T^f_{p_j}$ with $p_j \in \tau - \tau_a$ one has that $\nu^{\varepsilon}_{\tau}(T^f_{p_j}) = 2n-2$. If, on the other hand, R_f is such that for every trajectory T^f_p with $p \in \tau$ one always has $\nu^{\varepsilon}_{\tau}(T^f_p) = 2n$, then R_f is said to be n-dimensional without boundaries. The definition of reference frame we have given above is different from that given in a previous paper. Yet, these two definitions share a certain number of properties that are characteristic of reference frames (García-Sucre, 1978a). We prefer the present definition because it is simpler and, in our view, corresponds better to the way in which reference frames are established in practice, although with both definitions we can associate unambigously space and time coordinates to points of particle trajectories.

The present definition of reference frame accounts for the following features: (a) the points of a reference frame R_f are points belonging to the space–time $f = ST(S)$ in which R_f is defined; (b) the way in which the points of R_f distribute themselves in R_f depends upon the choice of physical systems $S_1, \ldots, S_n$ included in S such that the sets τ and ε corresponding to R_f are constructed with evolving particles belonging to $S_1 \cup \cdots \cup S_n$.

We illustrate in Figure 1 a reference frame R_f in a $f = ST(S) = \Sigma\Sigma(S)/\sim$ such that the corresponding set ε has a unique element p' and $\tau = \{p_1, \ldots, p_9\}$. The slanted arrow in the figure together with all those double circles crossed over by this arrow represent the trajectory $T^f_{p'}$, where $p' \in \varepsilon$. Each vertical arrow together with those double circles over which it passes represent the trajectory $T^f_{p_i}$ of a particle $p_i \in \tau (i = 1, 9)$. Double circles stand for the points of $f = \Sigma\Sigma(S)/\sim$ that are points of R_f. Therefore, these double circles represent equivalence classes of points of crossing belonging to $\Sigma\Sigma(S)$. Each of these points of R_f can be differentiated from any other point of R_f because each of these points has a different structure. The way in which the points of each trajectory $T^f_{p_i}$ with $p_i \in \tau$ appear in the figure depends upon the ordering of the α states of p_i. The ordering of the points of each trajectory is specified by the arrows in the figure. The way in which the trajectories $T^f_{p_i}$ are horizontally ordered in the figure is fixed by the ordering of the α states of p'.

Vertical trajectories in this figure can be interpreted as clocks of the reference frame R_f. The one-to-one mappings χ between pairs of trajectories of particles belonging to τ which preserve the order in each of these trajectories can be used to assign time coordinates to the points of R_f. For

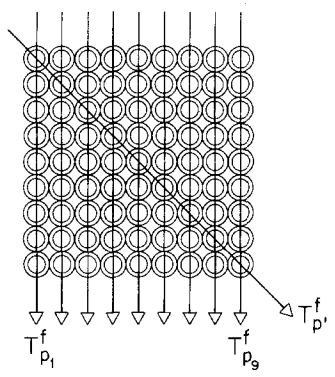

Fig. 1. We illustrate here a reference frame $R_f=(R_f^0;\tau;\varepsilon)$. Double circles stand for the points of the field $f=\Sigma\Sigma(S)/\sim$ that also belong to R_f^0. Each vertical arrow together with those double circles over which it passes represent the trajectory $T^f_{p_i}$ of an evolving particle $p_i\in\tau(i=1,9)$. The slanted arrow together with all those double circles crossed over by this arrow represent the trajectory $T^f_{p'}$ of an evolving particle $p'\in\varepsilon$. The case illustrated here corresponds to a set ε with only one particle, and to an order of immediate connection $\nu^e_\tau(T^f_{p_i}),p_i\in\tau$, equal to either 1 or 2. Then R_f is a one-dimensional reference frame with a zero-dimensional boundary.

each pair of trajectories $T^f_{p_i}$ and $T^f_{p_j}$ with $p_i,p_j\in\tau$ there exists only one possible one-to-one mapping χ_{ij} that preserves the order of the points of these trajectories. With all such mappings χ_{ij} we put into correspondence all the first elements of the trajectories $T^f_{p_i}$, where $p_i\in\tau$. In the same way, we put into correspondence the second elements of the same trajectories, the third elements, etc..... Then, we assign the same time coordinate to those points of R_f that correspond to each other according to the above mappings χ_{ij}. Finally, choosing a point of R_f as the origin we assign time and spatial coordinates to each point of R_f by respectively counting the number of horizontal rows of points and vertical trajectories separating each point of R_f from the origin (see Figure 1).

For the sake of simplicity, the reference frame represented in Figure 1 has been chosen to be such that the corresponding set ε has a unique element: the particle p'. According to our definition of reference frame, we can also have examples of reference frames for which the sets ε are such that more than one particle belongs to them. Furthermore, our example is simple enough to be such that the order of immediate connection $\nu^e_\tau(T^f_{p_i})$ of trajectories $T^f_{p_i}$ with $p_i\in\tau$ is equal to either 2 or 1. Then, R_f in Figure 1 is a one-dimensional reference frame with a zero-dimensional boundary. The space–time diagram associated with this reference frame has only one bounded space axis and one bounded time axis. Furthermore, both these

axes are discrete. In general, we can have reference frames with more than one space axis. Note that space–time diagram and space–time are completely different concepts. The first one is simply a system of space and time coordinate axes which is associated to a given reference frame (Taylor and Wheeler, 1966, Chap. 1), while a space–time is a physical entity embracing reference frames and physical systems.

Given an arbitrary particle p_k whose trajectory $T^f_{p_k}$ in the space–time $f = ST(S)$ is such that $T^f_{p_k} \cap R_f \neq \varnothing$, then to every point of $T^f_{p_k}$ that also belongs to R_f can be associated space and time coordinates in R_f. We have already defined what we understand by trajectory of a particle in a field and thus also in a space–time. Now we define trajectory of a particle in a reference which must be clearly distinguished from the first. In fact, we have used trajectories of particles in a field to define reference frames in that field, and we define presently trajectories of particles in a reference frame by using this last concept.

Given a particle p and a reference frame R_f in $f = ST(S)$, the trajectory $T(p; R_f)$ of p in R_f is a set having the same elements as T^f_p; but this set differs from T^f_p in that the ordering of the points of $T(p; R_f)$ is induced by the time axis of R_f instead of the set $\Sigma(p)$ of α states of p, according to the rule: for two $x, y \in T(p; R_f)$ one has that $x \prec y$ if the time coordinate of x in R_f is smaller than that of y (García-Sucre, 1978b).

Following the above definition, $T(p; R_f)$ may be either a partly or a completely ordered set depending on whether some of its points have the same time coordinates in R_f or not. Furthermore, the ordering in $T(p; R_f)$ does not depend upon the fact that p is either an evolving or a nonevolving particle, since the order of the elements of $T(p; R_f)$ is induced by the time axis of R_f (García-Sucre, 1978b). In the same way we can define the trajectory $T(S_i; R_f)$ of the physical system S_i in the frame R_f as the set having the same elements as $\cup_{p \in S_i} T^f_p$ and ordered according to their time coordinates in R_f.

Given a space–time $ST(S) = \Sigma\Sigma(S)/\sim$, such that S represents a physical system to which a very large number of particles belong, then there exist many different reference frames in the same space–time, each one corresponding to a different collection of systems $S_1, \ldots, S_n$ included in S. It may even occur that more than one frame extending over the whole of a space–time exists. This occurs when every pair of points of a space–time $ST(S)$ are connected by many evolving particles of S, and so we can choose different pairs of sets τ and ε, each pair giving rise to a frame having as points all those belonging to $ST(S)$. This is related to the fact that to specify a frame in a given spacetime $ST(S)$ consists in selecting one of the many ways in which points belonging to $ST(S)$ are connected to each other by evolving particles belonging to S. Thus, even if a frame

$R_f = (R_f^0; \tau, \varepsilon)$ has all the points of $ST(S)$, i.e., $R_f^0 = ST(S)$, this frame may correspond to one of the possible selections of sets τ and ε included in S (see the definition of reference frame).

A reference frame R_f in a spacetime $f = ST(S)$ is said to be inertial if the intensity of f has the same value at every point of R_f.

We will see in the next section that this definition of inertial frame is consistent with the constancy of the wavelength and frequency of a physical system when it travels from one region to another of the same inertial frame. Now, as the last point of this section, let us discuss the relation existing between the above definition and the usual way in which inertial frames are characterized.

Assume that the space–time $f = ST(S)$ under consideration is nonhomogeneous. Therefore, any reference frame R_f covering a sufficiently large number of points belonging to $ST(S)$ will be manifestly nonhomogeneous. If R_f is such that the intensity of f changes gradually between neighboring points on R_f, it is clear that as we consider frames covering only parts of R_f the nonhomogeneity of these frames will be, according to our definition, less manifest than for R_f. And this is what usually occurs when in order to get inertial frames in general relativity we are forced to work with small reference frames in both spatial distances and intervals of time that can be defined within these frames (see for instance Taylor and Wheeler, 1966, Chap. 1).

Let A be a region localized in space (e.g., a narrow stripe along a time axis) in the nonhomogeneous frame R_f where the intensity of f is considerably larger than that in any other region located sufficiently far from A in R_f. According to our definition of trajectory $T(p; R_f)$ it follows that points of trajectories of particles p will tend to appear more frequently in the region A than in any other region of R_f (this can be understood by recalling that a particle p will pass over those points of a field f with which it shares at least a preparticle, and that the larger the intensity of f in a point more preparticles will enter in this point). Since R_f is nonhomogeneous it is not an inertial frame according to our definition. Neither is R_f an inertial frame in the usual sense because trajectories $T(p; R_f)$ are deformed towards a region A of R_f localized in space. In this sense, there exists a parallelism between the usual definition of inertial frame and ours. Furthermore, we can follow steps corresponding well with those of the usual procedure in order to get an inertial frame within a prefixed degree of approximation (Taylor and Wheeler, 1966, Chap. 1). In our example above we can define a new reference frame R_f' having part of the points of R_f and differing from R_f in the sets τ and ε, i.e., $R_f' = (R_f'^0; \tau' \varepsilon')$, where $R_f'^0 \subset R_f^0, \tau' \neq \tau$ and $\varepsilon' \neq \varepsilon$. Moreover, each particle belonging to τ' is chosen in such a way that its trajectory approaches as closely as possible the average trajectories of particles passing over the same region of R_f.

Disregarding oscillations around mean values we obtain in this way a reference frame which may be taken as inertial in the usual sense of the term (Taylor and Wheeler, 1966, Chap.1). Obviously, in this case, as when we are concerned with the usual definition of inertial frame (Taylor and Wheeler, 1966, Chap. 1), once we have fixed the accuracy within which a frame R_f' is inertial, this determines the maximal extension that R_f' may have beyond which R_f' cannot be inertial within the prefixed accuracy in question. To see this in terms of the elements of our model, assume that R_f' is a rather extended frame both in space and time, i.e., that considerably more than one particle belongs to τ' and that each of these particles has much more than one α state. For the sake of simplicity, assume that both R_f and R_f' are frames of the same kind as the frame illustrated in Figure 1. The frame R_f' has been constructed in such a way that it covers a given curved stripe of points of R_f. If the particles belonging to τ' tend to approach the region A of R_f, then they come closer to each other as they approach the region A. Thus, the curved stripe of points of R_f covered by R_f' should be spatially larger far from A than near to A. But this is not compatible with the equidistance between the points of the trajectories of the particles belonging to the set τ of a frame such as those illustrated in Figure 1. This feature has been represented in this figure by choosing vertical lines that represent particles belonging to τ parallel to each other. And this corresponds in the usual operational way of defining inertial frames to the fact that concrete inertial frames are considered to be rigid bodies in the sense that they do not adapt themselves to the shape of the line of forces of the gravitational field under consideration (Taylor and Wheeler, 1966, Chap. 1). This problem of reference frames in a nonhomogeneous space–time will be further analyzed in a forthcoming paper (García-Sucre, 1979b).

3. ENERGY, FREQUENCY, MOMENTUM, AND WAVELENGTH OF A PHYSICAL SYSTEM IN A REFERENCE FRAME

We need a statistical description of reference frames in order to describe what we understand by a detector of particles, and frequency, and wavelength of a particle in a reference frame.

A reference frame R_f is a *replica* of R_f', if the following conditions are fulfilled:

(i) $N[R_f^0] = N[R_{f'}^{\prime 0}]$, where $R_f \equiv (R_f^0; \tau; \varepsilon)$ and $R_f' = (R_{f'}^{\prime 0}; \tau'; \varepsilon')$.

(ii) given a point x of R_f and a point y of $R_{f'}'$ such that x has in R_f the same coordinates as y in $R_{f'}'$, then x and y fulfill that $I(x;f) = I(y;f')$.

Note that in the above definition that generally $f \neq f'$; however, it also covers the particular case in which $f = f'$. On the other hand, the preparticles entering in the points of any given reference frame necessarily belong to the set B, since by definition B is the set whose elements are all the preparticles. Furthermore, in our formalism a preparticle is equally suitable to enter in a point of a given frame just like any other preparticle of B, since one has not stated any condition about the possibility of preparticles entering or not in the points of a reference frame. Therefore, if we consider statistical ensembles of reference frames using the above criterion of replica, we have the following property: the probability $\rho(x_i; R_f)$ that a given preparticle enters in the point $x_i \in R_f^0$ is given by the ratio

$$\rho(x_i; R_f) = I(x_i; R_f)/N[B] \tag{3.1}$$

where $I(x_i; R_f)$ is the intensity of the field f at the point $x_i \in R_f^0$. We denote as $\rho(R_f)$ the average of $\rho(x_i; R_f)$ over the points $x_i \in R_f$, i.e.,

$$\rho(R_f) \equiv \frac{1}{N[R_f]} \sum_{i=1}^{N[R_f]} \rho(x_i; R_f) \tag{3.2}$$

In the same way we have the average intensity of f at the points of R_f:

$$I(R_f) \equiv \frac{1}{N[R_f]} \sum_{i=1}^{N[R_f]} I(x_i; R_f) \tag{3.3}$$

If R_f is an inertial frame, it follows from the definition of inertial frame and equations (3.1)–(3.3) that $\rho(R_f) = \rho(x_i; R_f)$ and $I(R_f) = I(x_i; R_f)$ for every $x_i \in R_f$.

According to equation (3.1) there is a certain probability different from zero that a preparticle of S_i may enter in a given point of R_f. Therefore, if the frame R_f is such that $N[R_f]\rho(R_f) \gg 1$ and $N[R_f] \gg \nu\pi(S_i)$, where $\nu\pi(S_i)$ denotes the total number of preparticles belonging to the elements of the particles that belong to S_i, then it may occur that all the preparticles of S_i enter in the points of R_f. Since the same preparticle may appear in several points of R_f it may even occur that considering only a part of the points of $T(S_i; R_f)$ all the preparticles of S_i already enter in these points. To see that a same preparticle of S_i may enter in several points of R_f recall that according to our Postulate 1 the points of a field are distinct from each other only because these points have different structures. Then, taking into account that particles entering in different points of crossing are such that their α states may intersect each other, it follows that the same preparticle may appear in different points of the same field.

Thus, the probability $\rho(x_i; R_f)$ that a preparticle enters in the point x_i of R_f is independent of the fact that the same preparticle enters or not in another point of R_f.

When $N[R_f]\rho(R_f) \lesssim 1$ it may occur, in contrast with the case described above, that only a part and eventually none of the preparticles of S_i enter the points of R_f.

We can choose a unit interval of time in terms of which the intervals in any trajectory $T_p^f, p \in \tau$, can be measured. This can be done by fixing the number or points along any trajectory T_p^f that are covered by a unit interval of time. At least it is otherwise explicity indicated we fix each unit interval of time to cover only one point of R_f.

The *frequency of a physical system S_i in an inetial frame R_f* is the number of points belonging to $T(S_i; R_f)$ that also belong to an arbitrary trajectory $T_p^f, p \in \tau$, per unit interval of time. Let us denote this frequency as $\Omega(S_i; R_f)$.

In general, the frequency $\Omega(S_i; R_f)$ may not have a well-defined value. The concepts of mean frequency $\bar{\Omega}(S_i; R_f)$ and standard deviation $\Delta\Omega(S_i; R_f)$ then become necessary. To calculate $\Omega(S_i; R_f)$ divide $N[T(S_i; R_f)]$ by the total number of intervals necessary to cover all the trajectories $T_p^f, p \in \tau$, of the frame R_f. The frequency of a system S_i in a frame R_f has a well-defined value if $\Delta\Omega(S_i; R_f) = 0$. If we are concerned with a physical system S_k to which only one particle p_k belongs then we speak interchangeably of the frequency $\Omega(S_k; R_f)$ of the system S_k or the frequency of the particle p_k in the frame R_f, which we denote $\Omega(p_k; R_f)$.

Consider a system $S_i = \{p_i\}$, where $p_i = \{\{\alpha_e\}\}$ is a particle in which only one preparticle enters. From equations (3.1) and (3.2) it follows that the frequency $\Omega(p_i; R_f)$ of p_i in R_f fulfills the relation

$$\Omega(p_i; R_f) = \rho(R_f) \tag{3.4}$$

Note that according to this equation and equation (3.1) it follows that $0 \leqslant \Omega(p_i; R_f) \leqslant 1$, which is a consequence of the fact that we have taken each unit interval of time as covering only one point of R_f.

In the case of a particle p_j in which many preparticles enter the frequency $\Omega(p_j; R_f)$ fulfills the relation $\Omega(p_j; R_f) = g(\nu\pi(p_j))\rho(R_f)$, where $\nu\pi(p_j)$ stands for the number of preparticles participating in p_j. An expression for the function $g(n)$, where $n \equiv \nu\pi(p_j)$, can be obtained by considering the equation

$$\begin{aligned} N[T(p_j; R_f)] = {} & \rho(R_f)N[R_f] + \rho(R_f)\{N[R_f] - \rho(R_f)N[R_f]\} \\ & + \rho(R_f)\{N[R_f] - \rho(R_f)N[R_f] - \rho(R_f)(N[R_f] - \rho(R_f)N[R_f])\} \\ & + \cdots + (n\text{th term}) \end{aligned} \tag{3.5}$$

The first term on the right-hand side of equation (3.5) is the number of points of R_f in which a preparticle entering in p_j appears. The second term on the right-hand side of equation (3.5) is the number of points of R_f in which a preparticle of p_j appears, once those points of R_f corresponding to the first preparticle of p_j that we have already considered are discarded. The third term in the above summation is the number of points of R_f in which a preparticle entering in p_j appears, once those points of R_f corresponding to either of the two preparticles already considered are discarded, and so on and so forth. In this way we take into account, first, that the same preparticle may enter in more than one point of R_f, and second, that if two or more preparticles of p_j enter in the same point of R_f, this point contributes as only one point in the counting of points of the trajectory $T(p_j; R_f)$.

Making use of the relation $\Omega(p_j; R_f) = N[T(p_j; R_f)]/N[R_f] = g(n)\rho(R_f)$, we obtain from equation (3.5)

$$g(n) = n - \frac{n(n-1)}{2}\rho(R_f) + \cdots + (-1)^{n-1}\rho^{n-1}(R_f) \tag{3.6}$$

The terms in the above equation in which $\rho^m(R_f)$ with $m > 1$ appears take into account that when $n = \nu\pi(p_j)$ can no longer be neglected vis-à-vis the total number of preparticles entering in the points of R_f, $\nu\pi(R_f)$, then the probability that more than one preparticle of p_j enter in the same point of R_f cannot be neglected either. Yet, in the most frequent case where $\nu\pi(p_j) \ll \nu\pi(R_f)$, the equation (3.6) reduces itself to $g(n) = n$, and then in this case $\Omega(p_j; R_f) = \nu\pi(p_j)\rho(R_f)$.

Similarly, for a physical system S_i in which $\nu\pi(S_i)$ preparticles enter, we can write

$$\Omega(S_i; R_f) = g(\nu\pi(S_i))\rho(R_f) \tag{3.7}$$

The *energy of a physical system S_i in an inertial frame R_f* is equal to the number of points of the trajectory $T(S_i; R_f)$ of S_i in R_f. Denoting this energy as $E(S_i; R_f)$ we may otherwise write

$$E(S_i; R_f) \equiv N[\, T(S_i; R_f)\,] \tag{3.8}$$

When only one particle belongs to S, say p_m, we speak interchangeably of the energy $E(S_i; R_f)$ of the system $S_i = \{p_m\}$ or the energy $E(p_m; R_f)$ of the particle p_m in the frame R_f.

Let $v(S_i; R_f)$ denote the magnitude of the velocity of the system S_i in the frame R_f. We calculate $v(S_i; R_f)$ from the trajectory of S_i in R_f in the following way. Recall that $T(S_i; R_f)$ is ordered according to the time

coordinates in R_f of the points belonging to $T(S_i; R_f)$. As we have already explained, $T(S_i; R_f)$ may be either a partly or completely ordered set, depending on whether some of its points have equal time coordinates in R_f or whether, on the contrary, to each point of $T(S_i; R_f)$ corresponds a different time coordinate in R_f. To calculate $v(S_i; R_f)$ let us consider the graph whose points are all the points belonging to $T(S_i; R_f)$, and the oriented lines (arrows) joining points of $T(S_i; R_f)$ occur only between two consecutive points. Furthermore, each arrow between two points ends at the point with the larger time coordinate. For instance, suppose that all the points of the trajectory $T(S_i; R_f)$ are ordered according to their time coordinates in R_f. Then the graph of the trajectory $T(S_i; R_f)$ will be a one-branch-oriented graph such that there exists only one path of consecutive arrows that covers completely the graph in question. If, on the other hand, there exists a subset t of points of $T(S_i; R_f)$ having the same time coordinate in R_f, then the graph of $T(S_i; R_f)$ must be ramified in the region where the points belonging to t appear. For instance, suppose that $t = \{x_i, x_j\}$ and that $x_a \in T(S_i; R_f)$ and $x_b \in T(S_i; R_f)$, respectively, immediately precedes and succeeds x_i and x_j in the graph of $T(S_i; R_f)$, and that there exists only one such subset t of $T(S_i; R_f)$. Then the graph of $T(S_i; R_f)$ will have only one branch from the first point of $T(S_i; R_f)$ to the point x_a; after that it becomes ramified with two branches passing, respectively, over x_i and x_j, immediately these two branches join together at x_b; thereafter the graph of $T(S_i; R_f)$ has only one branch up to the last point of $T(S_i; R_f)$.

To calculate the magnitude $v(S_i; R_f)$ of the velocity of the system S_i in the frame R_f we ascribe to each arrow of the graph of $T(S_i; R_f)$ a number obtained by dividing the spatial increment by the time increment existing between the two points joined by the arrow under consideration in the frame R_f, and then we average all these numbers. On the other hand, the direction of the vector $\mathbf{v}(S_i; R_f)$ can be found by using the same graph of $T(S_i; R_f)$ as we have already used to calculate the magnitude of $\mathbf{v}(S_i; R_f)$. This can be done by considering the projections of the oriented segments of the graph of $T(S_i; R_f)$ along the space axes associated to R_f. Each of such projections can be considered as a vector whose direction is defined by the order of the two points of $T(S_i; R_f)$ determining the oriented segment under consideration. The direction of $\mathbf{v}(S_i; R_f)$ will be then given by the direction of the vector sum of all such vectors.

In this way, we ascribe a unique mean velocity to S_i in the frame R_f. This way of proceeding corresponds well to the usual prescription to calculate mean velocities, as becomes apparent in the case where the graph of $T(S_i; R_f)$ appears in R_f as a smooth curve. Yet, our prescription also allows us to handle cases where the graph of $T(S_i; R_f)$ appears very intricate due to the number and the dispersion of the points of $T(S_i; R_f)$ in

R_f, and/or the ramifications that such a graph may present. We are now prepared to define what we understand by the mass of a physical system S_i.

We say that *a physical system S_i is at rest in an inertial frame R_f* if $v(S_i; R_f) = 0$.

The *mass of a physical system* S_i is the number of points of the trajectory $T(S_i; R_f)$ of S_i in the inertial frame R_f where S_i is at rest.

Note that we have not used above the term rest mass, but only mass, since each time that we will speak of mass it will be understood that we are considering the number of points of the trajectory of a system in a frame where this system is at rest.

According to the above definitions, it follows that the energy of a system in the frame where it is at rest is equal to the mass of the same system. As is well known, this is a feature characteristic of relativity (see, for instance, Taylor and Wheeler, 1966, Chap. 2). We will go into a further discussion of the relation between energy and mass at the end of Section 4.

From the above definitions of energy and frequency of a physical system in a reference frame it follows immediately that there exists a proportionality relation between energy and frequency of a physical system. From equations (3.7) and (3.8) we can write

$$E(S_i; R_f) = g(\nu\pi(S_i))\rho(R_f)N[R_f] \tag{3.9}$$

which, using again equation (3.7), can also be written in the form

$$E(S_i; R_f) = N[R_f]\Omega(S_i; R_f) \tag{3.10}$$

Note that the function $g(n)$ in equation (3.6) does not appear in the above proportionality relation between energy and frequency since it is absorbed by the term $\Omega(S_i; R_f)$ in equation (3.10). Therefore, the specific form of $g(n)$ given in equation (3.6) does not intervene in the above important proportionality relation of our model. The same will occur below with the proportionality relation between energy and frequency when both are defined locally [see equation (3.11)].

Note in relation to equation (3.10) above that according to our definition of inertial frame the frequency $\Omega(S_i; R_f)$ will not change when we pass from one region of R_f to another. This is a consequence of the fact that by our definition of inertial reference frame the intensity $I(x; f)$ has the same value for any point x of R_f, and therefore the probability $\rho(x_i; R_f)$ that a given preparticle enters in a point x_i of R_f is the same for any other point of R_f. In practice we cannot choose a reference frame such that the number of preparticles entering in each point of R_f is exactly the same. However, we can tend to this ideal by choosing reference frames such that

—although the number of preparticles entering in each of their points is not exactly equal to each other—by partitioning R_f in equal and sufficiently large regions we can mask these fluctuations in the intensity $I(x; R_f)$, where $x \in R_f$. A frame R_f will be nearer to our ideal of inertial reference frame inasmuch as these regions in which R_f is partitioned can be of a smaller extension. The complementary case occurs when systematic drifts of the mean number of preparticles per field point appear as we pass from one region to another, i.e., when systematic drifts in the mean intensity $\overline{[I(x_i; R_f)]}_{x_i \in R_{\tilde{f}}}$ occur as we pass from one region $R_{\tilde{f}}$ to another region $R_{\tilde{f}'}$ of R_f. In this case, it is necessary to define a frequency and an energy for each region $R_{\tilde{f}}$ of R_f. This will be done below in the paragraph preceding equation (3.11).

There is another aspect in which noninertial frames play an important role in our model. We have defined a trajectory $T(S_i; R_f)$ as an ordered set of points of R_f. However, we have not mentioned the effect that the system S_i may have on the frame R_f, an effect arising due to the interaction between $f = \Sigma\Sigma(S)/\sim$ and $f_i = \Sigma\Sigma(S_i)/\sim$ [i.e., because $f \cup f_i \neq \Sigma\Sigma(S \cup S_i)/\sim$], f being the field where R_f is defined. In fact, we have until now implicitly assumed that the system S_i is already included in S and that R_f still remains an inertial frame.

To understand the effect that S_i may have in R_f consider the field $f' = \Sigma\Sigma(S \cup S_i)/\sim$. It is clear that f and f' may be either very similar or moderately different or still very different from each other depending on S and S_i. In connection with reference frames in each of these fields, we adopt the criterion that the frame in f' most closely related to R_f is the frame $R_{f'}$ determined in f' by the same sets τ and ε that determine R_f in f. This is always possible since according to our definition of reference frame one has that $\varepsilon, \tau \subseteq S \subseteq S \cup S_i$. Then, the question arises as to which of the two frames R_f or $R_{f'}$ is inertial. If we start with an inertial frame R_f we will very probably end with a frame $R_{f'}$ that is noninertial because of the effect of S_i on R_f. As we have already discussed at the beginning of this section, the field f may be affected in several ways by its interaction with the field $f_i = \Sigma\Sigma(S_i)/\sim$ to yield the field $f' = \Sigma\Sigma(S \cup S_i)/\sim$. In this sense, it may be expected that because of the redistribution of the points of crossing in different equivalent classes when we pass from R_f to $R_{f'}$, the intensity of f' will be for some points larger and for some other points smaller than the uniform intensity of f in the points of R_f (see the definition of intensity of a field in a point of a reference frame). This can be understood recalling the way in which each point of the trajectory $T(S_i; R_{f'})$ arises. Note that now in this trajectory $T(S_i; R_{f'})$ we have $R_{f'}$ instead of R_f and that although S_i is again assumed to be included in the physical system producing f' we open the possibility that $R_{f'}$ may be noninertial (and this corresponds well with

the usual situation in which it is due to the "presence" of a physical system in a frame that this frame loses its inertial character and also that this physical system manifests itself in the frame). Recall that a point x' of the trajectory $T(S_i; R_{f'})$ arises because at least one particle of S_i crosses over the center of a point of crossing belonging to x' (an equivalence class of similar points of crossing) of f'. In order to describe the noninertiality of $R_{f'}$ due to the interaction between f and f_i yielding f', let us start from the inertial frame R_f. Assume that $p \in S_i - S$ and that p crosses over the center of a point of crossing σ belonging to a point x of R_f. As a result, this point of crossing σ would become a point of crossing σ' with a different structure from σ (see the definition of a point of crossing and of its structure) and belonging to a point $x' \in R_{f'}$ which has a different structure from the point x of R_f. Then, if no other particle $p' \in S_i - S$ interferes with the points x and x' that we have just described, we can say that the point $x' \in T(S_i; R_f)$ is such that the intensity $I(x'; f')$ of f' in x' is larger than $I(x; f)$. Furthermore, since R_f is an inertial frame it follows that $I(x'; f')$ is larger than the intensity of f in any point of R_f if we neglect small fluctuations of the intensity of f when we change points in R_f. In the same way, there would be a point $x'' \in R_{f'}$ assembling the remainder points of crossing of x once x has lost the point of crossing σ. The point x'' should be such that $I(x''; f')$ is smaller than the intensity of f in any point of R_f if we again neglect small fluctuations. Then, if both the number of particles of S_i and the number of preparticles entering in each of them are small in comparison with those of R_f, we may expect that for each point of $T(S_i; R_{f'})$ there exists a point of $R_{f'}$ where the intensity is larger than the mean intensity of f' in $R_{f'}$, and another point where it is smaller. Although even in this simple case things are much more chaotic than what happens with the regular oscillations of a moderately intense (electromagnetic) field, we will associate with each point of a trajectory $T(S_i; R_{f'})$ an oscillation of f'. The situation becomes more complicated when the above condition as to the "smallness" of S_i with respect to R_f is not fulfilled, since in this case the action of a particle of S_i on R_f may very probably be either partly or completely destroyed by another particle of S_i. Because of the above correspondence between points of $T(S_i; R_{f'})$ and "oscillations" of f', and the fact that for a regular distribution of points of $T(S_i; R_{f'})$ along a time axis of $R_{f'}$ periodicity manifests itself when we pass from one to another point of $T(S_i; R_f)$, we will associate cycles with points of trajectory along time axes in our discussion below concerning systems of units.

Despite its stark simplicity, we should give credit to our model and try to interpret the factor $N[R_f]$ of proportionality between energy $E(S_i; R_f)$ and frequency $\Omega(S_i; R_f)$ in equation (3.10) as the Planck constant expressed in the system of units we have been using. Recall that this system of units

is such that energy, mass, spatial distances, and intervals of time are each expressed as a number of field points.

To this end, we first adopt the usual convention that all the R_f's that we consider as reference frames must have the same number of points. As is well known, this is fulfilled by all the frames involved in Lorentz transformations. The same applies to the case of Galilean transformations since both these transformations establish a one-to-one correspondence between the points of different frames. Second, let us denote as N_0 the total number of points of a field f extending over the whole universe, i.e., $N_0 \equiv N[f]$. Third, consider a frame $R_{\tilde{f}}$, where $\tilde{f} \subset f$, covering a given restricted region of consecutive points of the reference frame R_f defined in the whole field f, i.e., $N_0 \equiv N[R_f]$ and $N[R_{\tilde{f}}] < N_0$. Fourth, in a similar way as equation (3.10) holds for the entire reference frame R_f, we can now consider the number $N[R_{\tilde{f}}]\Omega(S_i; R_{\tilde{f}})$ of points of the trajectory $T(S_i; R_f)$ found in the restricted region $R_{\tilde{f}}$ of R_f. Finally, we ascribe to the region $R_{\tilde{f}}$ of R_f the energy that S_i would have in a fictitious reference frame R_f', having the same number of points as R_f, and such that the density with which the points of the trajectory of S_i appear in any region of R_f' is the same as in the restricted region $R_{\tilde{f}}$ of R_f. We can express this last point by means of the relation $E(S_i; R_f') = (N[R_f]/N[R_{\tilde{f}}])N[R_{\tilde{f}}]\Omega(S_i; R_{\tilde{f}})$, i.e.,

$$E(S_i; R_f') = N_0 \Omega(S_i; R_{\tilde{f}}) \tag{3.11}$$

Note in this equation that the frequency $\Omega(S_i; R_{\tilde{f}})$ is defined in the restricted region $R_{\tilde{f}}$ of R_f. Therefore, equation (3.11) is suitable for those cases in which we have systematic drifts in the mean frequency when we pass from one region to another in a reference frame R_f which is not inertial. To be sure, equation (3.11) reduces itself to equation (3.10) when such drifts are not present.

The fact that the number of N_0 appearing in equation (3.11) may be expected to be an extremely large number should not be too surprising since we have measured energy, frequency, and time in units completely different from those of the cgs system of units in which the Planck constant is usually expressed. To further analyze this question we will make an assumption on the nature of light which will lead us to assign the value 1 to the velocity of light c when both space and time separations are expressed as numbers of field points. (For arguments in favor of systems of units in which $c = 1$ of a different nature than those that we will give below, see Taylor and Wheeler, 1966, Chap. 1.) Before going into this question, and in order to have a rough idea of the relation that exists between the cgs system of units and the system of units in which energy, space, and time separations are expressed in numbers of field points, let us call N_{cm}

and N_s the number of field points covering respectively 1 cm along a row of points and 1 seg along a column of points of a reference frame represented with the same conventions as in Figure 1. Also let N_g be the number of field points corresponding to an energy (expressed in mass units) of 1 g. Then, comparing equation (3.11) with the equation $E=h\nu$ under the form

$$\frac{E}{c^2}N_g=\frac{h}{c}N_gN_{cm}\frac{\nu}{cN_{cm}} \tag{3.12}$$

we get $N_0=(h/c)N_gN_{cm}$. Assuming the mass of the universe $\sim 10^{55}$ g and its radius $\sim 10^{28}$ cm, we obtain the value $N_0\sim 10^{300}$, $N_g\sim 10^{245}$, $N_{cm}\sim 10^{72}$, and $N_s=cN_{cm}\sim 10^{82}$, all these numbers being expressed in field points. Below, we will come again to the relation $N_s=cN_{cm}$.

Briefly, if in our model we interpret equations (3.10) and (3.11) as equations of the type $E=h\nu$, this leads us to relate the Planck constant with the total number of field points that has a reference frame extending over the universe. In this view, the Planck constant is connected with a macroscopic property of the universe rather than with its structure at the microscopic level. Let us point out that our assumption according to which the number N_0 of points of a reference frame R_f extending over the whole universe is constant, is compatible with models of the universe in which its size changes with time (e.g., the model of the expanding universe). This compatibility is based on the fact that the change in size of the universe with time can be translated in our terms as a change of the maximal space separation that can be defined within R_f as we consider different lapses of time within the same R_f. While, on the other hand, the constancy of N_0 refers to the fact that the total number of points of R_f (i.e., taking into account all possible space separations and time durations within R_f) is constant.

Consider a frame R_f whose space–time diagram only has one space axis, as in the case illustrated in Figure 1. For arbitrarily chosen physical systems S_i, the most probable distribution of the points of trajectories $T(S_i;R_f)$ in the frame R_f will be those in which the points of the trajectories are uniformly distributed in R_f. This results from the fact that what is important to the location in R_f of a point of a given trajectory $T(S_i;R_f)$ is that the point of R_f in question shares at least one preparticle with S_i. And this does not depend upon the way in which the points of R_f are connected to each other, which is the trait distinguishing time and space separation in our definition of reference frame. In this sense, recall that time separations are measured along trajectories $T_p^f, p\in\tau$, and space separations are measured along rows of points, each one belonging to a different trajectory

$T_p^f, p \in \tau$. Thus, the points of a trajectory $T(S_i; R_f)$ of a sufficiently large and randomly chosen system S_i are not distributed preferently along either the space of the time axes. This leads to the result that the mean velocity produced by all those points of $T(S_i; R_f)$ corresponding to displacements in one direction will be unity.

More generally, consider a region R_f' of a frame R_f and let the space–time diagram associated to R_f' be such that it necessarily has one time axis and at least one space axis, each of these axes covering the same number of points of R_f. In the particular case of R_f illustrated in Figure 1 the region R_f' of R_f fulfilling the above conditions necessarily has one time axis and one space axis.

We assume that *light in a region R_f' of R_f* is a physical system S_i such that the points of the subset $T(S_i; R_f') \subseteq T(S_i; R_f)$ are randomly distributed in the region R_f' of R_f. [For a definition of light see a previous paper in which we discuss some properties related to the invariance of the velocity of light (García-Sucre, 1978b).] This is clearly an oversimplified description of light since we have mentioned only one trait which cannot by itself characterize an electromagnetic radiation field. However, we are assuming that the above description of light does point to a property of light, this being the only one that proves relevant to our discussion. This recalls what occurs with special relativity, where the only property of electromagnetic radiation playing an essential role in the elaboration of the mechanics of neutral bodies is the invariance of the velocity of light. On the other hand, we will see below that the property that we have ascribed to light in connection with the region R_f' of R_f above leads to the invariance of the velocity of light provided that we interpret this velocity as a mean value. With this interpretation, if R_f' is small enough so that the number of points of the trajectory $T(S_i; R_f)$ falling in the region R_f' is not too large, then fluctuations of the velocity may arise. This means that depending on the number of preparticles entering in the system S_i we can always choose regions R_f' sufficiently small so that fluctuations of the velocity around the mean velocity corresponding to a large region of R_f become sizable. In this sense, although our model stipulates that the velocity of light is invariant if sufficiently large regions of R_f are considered, fluctuations around this mean velocity may be expected when we are concerned with sufficiently small regions of R_f. Experiments to measure the velocity of light using both a very low intensity (García-Sucre, 1978b) and a very low frequency electromagnetic radiation could be a test for our assumption on the nature of light.

In the examples under consideration, R_f only has one space axis and, therefore, the velocity $\mathbf{v}(S_i; R_f)$ will be a vector of magnitude $v(S_i; R_f)$ along the unique space axis of R_f. Furthermore, given that the points of the

trajectory $T(S_i; R_f)$ are randomly distributed in the whole of R_f, the magnitude $v(S_i; R_f)$ vanishes because the left-to-right velocity contribution to $v(S_i; R_f)$ is compensated by the velocity contribution in the opposite direction along the unique space axis of R_f. (A similar situation occurs in the case of an isotropic radiation field in which the wave-front velocity in one direction is canceled by the wave-front velocity in the opposite direction.)

Consider now two inertial frames R_f and $\tilde{R}_f$ with associated space–time diagrams as illustrated in Figure 2. The axes x, t and x', t' in the figure correspond to the space–time diagrams associated to R_f and $\tilde{R}_f$, respectively. Let $T_+(S_i; R_f)$ be the subset of $T(S_i; R_f)$ in which the order of the elements of $T(S_i; R_f)$ is preserved [i.e., $x,y \in T_+(S_i; R_f)$ and $x \prec y$ implies that $x \prec y$ in $T(S_i; R_f)$] and such that every pair of consecutive points of $T_+(S_i; R_f)$ corresponds to a positive displacement along the x axis. Similarly, one can define a $T_-(S_i; R_f)$ corresponding to negative displacements along the x axis. For random distribution of points the velocity $v_+(S; R_f)$ corresponding to $T_+(S_i; R_f)$ is equal to unity, since every pair of consecutive points of $T_+(S_i; R_f)$ defines a segment that determines an angle between 0 and $\pi/2$ with the x axis, and every such angle equally contributes to the mean value $v_+(S_i; R_f)$ when one averages all pairs of consecutive points of $T_+(S; R_f)$. Note that segments determined by consecutive points must be directed from the point with the smaller time coordinate to the point with the larger time coordinate. Let us emphasize that the angle between the x axis and any segment determined by two consecutive points of $T_+(S_i; R_f)$ cannot have a value between $\pi/2$ and π because in that case this pair of points will necessarily correspond to a

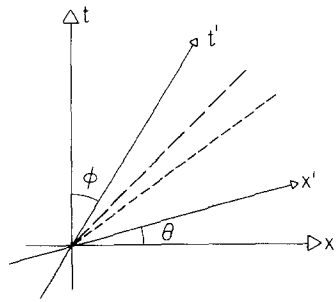

Fig. 2. The pairs of axes x,t and x',t' in this figure constitute the space–time diagrams associated with two one-dimensional reference frames R_f and $\tilde{R}_f$. The long track and shorter track dashed lines in this figure stand for trajectories having a well-defined velocity equal to unity ($c = 1$) in R_f and $\tilde{R}_f$, respectively. These two dashed lines do not coincide because in this figure $\theta \neq \phi$. In the case of special relativity $\theta = -\phi$.

negative displacement along the x axis. Thus, the directed segments determined by consecutive points of $T_+(S_i; R_f)$ have a direction necessarily comprised between the directions of the x axis and the t axis. The same applies to the space–time diagram with x' and t' axes. In this case, the directed segments between consecutive points of $T_+(S_i; \tilde{R}_f)$ have directions comprised between the directions of the x' axis and the t' axis, since any other direction would correspond to a negative displacement along the x' axis. Therefore, taking into account that the directions of the segments determined by pairs of points belonging to $T_+(S_i; \tilde{R}_f)$ are randomly distributed between the axes x' and t', it follows that also $v_+(S_i; \tilde{R}_f) = 1$ in the frame $\tilde{R}_f$. Note that the value $v_+(S_i; R_f) = 1$ does not correspond to the same direction than the value $v_+(S_i; \tilde{R}_f) = 1$ in Figure 2 where they have been respectively represented by the long track and shorter track dashed lines in this figure. These two directions in Figure 2 do not coincide because in this figure $\theta \neq \phi$. The points of the trajectory $T(S_i; \tilde{R}_f)$ of a system S_i having a well-defined velocity equal to unity in $\tilde{R}_f$ would appear all along a line bisecting the angle subtended by the x' and t' axes. Equivalently, the points of the trajectory $T(S_i; R_f)$ of a system S_i having a well-defined velocity equal to unity in R_f determine a line bisecting the angle subtended by the x and t axes. Therefore, for these two lines to coincide it is necessary that $\theta = -\phi$ in Figure 2. Although in our model all possible values of θ and ϕ are compatible with the invariance of the velocity of light when one interprets it as an average velocity, we will discuss below the particular case of $\theta = -\phi$ in order to make a comparison between our model and special relativity.

Following the above line of thought we then assume that the velocity of light in vacuum is equal to unity when both space and time intervals are expressed as numbers of field points. This assumption immediately leads to $N_s = c N_{cm}$, where N_s and N_{cm} are numbers of field points covered by intervals of 1 sec and of 1 cm, respectively, and c is the velocity of light in vacuum expressed in cgs units. In other words, the value $c \simeq 3 \times 10^{10}$ cm/seg could be interpreted within the framework of our model as a consequence of the fact that one second is much larger than one centimeter when both are expressed in terms of the same unit, namely a field point.

If we accept this conclusion, given that the choice of a system of units is a matter of convention, the feature mentioned above could be related to a property of living organisms, a particular class of which are humans, who have created according to their scale of distances and durations the known systems of units suitable for macroscopic physics. Only after the advent of quantum mechanics appropriate to describe atomic and subatomic physics, have other systems of units made their appearance. It is well known in this

concern that the system of units in which $c=\hbar=1$ is precisely one that leads to the stating of fundamental equations of physics in their simplest and most elegant form (see, for instance, Whichmann, 1971, Chaps. 2 and 5). This could be fortuitous; yet, from our point of view, it is a further indication that the convenience of fixing $c=1$ is rooted in something more profound than a simple convention. In connection with $\hbar=1$, recall that $\hbar=c=1$ allows us to express mass, energy, momentum, and frequency as the inverse of a distance. This can be expressed in our model by writing equation (3.10) in the form

$$\tilde{E}(S_i;R_f)=\tilde{\omega}(S_i;R_f) \tag{3.13}$$

where $\tilde{E}(S_i;R_f)=E(S_i;R_f)/\mu_d$, $\tilde{\omega}(S_i;R_f)=2\pi\Omega(S_i;R_f)\mu_d$, $N[R_f]/2\pi\mu_d^2=1$, and μ_d is a unit of both distance and time ($c=1$) covering a fixed number of field points. By taking $\hbar=1$ we then identify energies with frequencies, which come near to our definition of energy as a number of field points.

Let us go back to the question of why the Planck constant would have such an enormous value when expressed in our units, in contrast with the very small value that it effectively has in cgs units. From our point of view this can be understood as follows. When we use cgs units the left-hand side of the equation $E=h\nu$ is expressed in ergs, a unit corresponding to an already enormous number of field points. Therefore, the microscopic texture of the set $T(S_i;R_f)$ of field points as it appears in the frame R_f is hidden when $E(S_i;R_f)=N[T(S_i;R_f)]$ is expressed in ergs. Precisely the contrary occurs with the right-hand side of the same equation $E=h\nu$ when ν is expressed in terms of cycles per second. In this sense, recalling our reasoning in favor of a one-to-one correspondence between cycles and points of trajectories, the texture of $T(S_i;R_f)$ in R_f should be revealed all along an interval of time of a second, which, according to our previous discussion, covers an extremely large number of field points ($N_s=cN_c\sim 10^{82}$ field points). Therefore, the energies in cgs units may be expected to be numbers of very small magnitude in comparison with the corresponding frequencies expressed in cgs units, and this must be compensated by the constant h in the relation $E=h\nu$.

Let us now define what we understand by momentum of a physical system in an inertial frame. Let $\mathbf{v}(S_i;R_f)$ denote the velocity of the system S_i in the frame R_f.

The *momentum* $\mathbf{p}(S_i;R_f)$ *of* S_i *in an inertial frame* R_f is given by $E(S_i;R_f)\,\mathbf{v}(S_i;R_f)$.

From the fact that $E(S_i;R_f)$ is equal to the number of points of the trajectory $T(S_i;R_f)$, and from the way in which we have defined $\mathbf{v}(S_i;R_f)$, it follows that the momentum $E(S_i;R_f)\,\mathbf{v}(S_i;R_f)$ can be seen as the rate of

transport of the energy of S_i in R_f. It is well known that the relation between energy and momentum in relativity can be expressed under the form $p = Ev$ in a system of units where $c = 1$. This way of defining momentum is particularly suitable because of the intuitively clear relation between energy and momentum that $p = Ev$ immediately suggests (Taylor and Wheeler, 1966, Chap. 2).

Within the framework of our model we can give still another interpretation to the concept of momentum of a system S_i in an inertial frame R_f. First, consider a partition of trajectory set $T(S_i; R_f)$ in two sets $T_1(S_i; R_f)$ and $T_0(S_i; R_f)$ such that the points belonging to $T_1(S_i; R_f)$ contribute with the mean value 1 to the velocity $v(S_i; R_f)$, while those points belonging to $T_0(S_i; R_f)$ contribute with the mean value 0 to $v(S_i; R_f)$. We call $T_1(S_i; R_f)$ the *motion-set trajectory of* S_i *in* R_f. Then, it can be shown that $p(S_i; R_f)$ is precisely equal to the number of points of $T_1(S_i; R_f)$ for any physical system S_i such that $v(S_i; R_f) \leqslant 1$. In order to see this, we can write according to the definition of $T_1(S_i; R_f)$, the relation

$$\frac{1}{N_p} \sum_{i=1}^{N_p} v_i = 1 \tag{3.14}$$

where $N_p = N[T_1(S_i; R_f)]$ and every v_i corresponds to a pair of consecutive points of $T_1(S_i; R_f)$. We can similarly write

$$\frac{1}{N_E} \sum_{k=1}^{N_E} v_k = v(S_i; R_f) \tag{3.15}$$

where $N_E \equiv N[T(S_i; R_f)]$ and every v_k corresponds to a pair of consecutive points of $T(S_i; R_f)$. From equations (3.14) and (3.15) it follows

$$\frac{1}{N_E} \sum_{j=1}^{N_E - N_p} v_j + \frac{1}{N_E} \sum_{i=1}^{N_p} v_i = v(S_i; R_f) \tag{3.16}$$

where every v_j corresponds to a pair of consecutive points of $T_0(S_i; R_f)$. Recalling the definition of $T_0(S_i; R_f)$ according to which $\sum_{j=1}^{N_E - N_p} v_j = 0$, we obtain from equations (3.14) and (3.16) that

$$N_p = N_E v(S_i; R_f) \tag{3.17}$$

Therefore, from $N_p = N[T_1(S_i; R_f)]$ and our definition $\mathbf{p}(S_i; R_f) \equiv E(S_i; R_f)\mathbf{v}(S_i; R_f)$ we can finally write

$$p(S_i; R_f) = N\left[T_1(S_i; R_f)\right] \tag{3.18}$$

This property of our definition of momentum immediately suggests the following definition of wavelength of a physical system in an inertial frame:

The *wavelength* $\lambda(S_i; R_f)$ *of a physical system* S_i *in an inertial frame* R_f is equal to the mean distance in R_f between points belonging to the motion-set trajectory $T_1(S_i; R_f)$.

Here we will again have a mean value and a standard deviation for the wavelength and thus we will speak of well-defined wavelengths when the corresponding standard deviations vanish. Besides its intuitive character, the above definition has the virtue of leading immediately to a de Broglie type of relation between wavelength and momentum. This can be easily seen starting from the equation

$$\frac{N[T_1(S_i; R_f)]}{N[R_f]}\lambda(S_i; R_f) = 1 \tag{3.19}$$

which simply states that the probability $N[T_1(S_i; R_f)]/N[R_f]$ that an arbitrary chosen point of R_f may belong to $T_1(S_i; R_f)$, multiplied by the mean separation between points of $T_1(S_i; R_f)$ in R_f must yield unity. Therefore, from equations (3.18) and (3.19) we obtain

$$\lambda(S_i; R_f) = \frac{N[R_f]}{p(S_i; R_f)} \tag{3.20}$$

In the same way that equation (3.10) appears as the equation $E = h\nu$ by assuming $N[R_f] = N_0 = h$, where h is expressed as a number of field points, the above equation (3.20) appears as the de Broglie relation using the same assumption.

Let us remark here that according to our definitions of wavelength and inertial frame the mean wavelength does not change when we consider different regions of R_f, which is what we could expect to occur in inertial frames as they are usually defined. We have already made a similar remark in connection with frequency and inertial frame. On the other hand, our definitions of both frequency and wavelength can be generalized to the case of a noninertial frame, in which case frequency and wavelength should depend upon the region of the noninertial frame under consideration. This can be understood since noninertial frames are such that the number of preparticles entering in each of its points may change appreciably when we pass from one region to another of such a frame.

Until now in this section we have been mainly concerned with features of physical systems related to the delocalized aspect that they may

present in reference frames. For instance, we have defined frequency and wavelength of a physical system in a frame. Let us discuss at present how a physical system may be localized in a restricted region of a frame by the agency of a "detector" physical system.

Consider again the fields $f=\Sigma\Sigma(S)/\sim$ and $f'=\Sigma\Sigma(S\cup S_i)/\sim$ in which R_f and $R_{f'}$ are defined by the same pair of sets ε and τ. We say that S_i *is a detector physical system* if it fulfills the condition that when we pass from a frame R_f to a frame $R_{f'}$ there exists at least a physical system S_a, which not being localized in R_f is localized in $R_{f'}$ for a given interval of time larger than a prefixed threshold. This can be also expressed by saying that while the trajectory $T(S_a;R_f)$ is not spatially narrow in R_f for any interval of time larger than a prefixed threshold, the trajectory $T(S_a;R_{f'})$ does present such a narrow region in $R_{f'}$. Of course, in real situations we do not have the possibility of passing from R_f to $R_{f'}$ because we always are so to speak in $R_{f'}$ in which both the detector and detected systems are already included. As is usual, the frames R_f and $R_{f'}$ are used to indicate that $R_{f'}$ is appreciably different from R_f only in the region of $R_{f'}$ where either the detection of a given physical system (S_a in our example) takes place, or where noninertial effects such as forces manifest themselves, etc.

The way in which the system S_i behaves as a detector in the above description can be understood by the same kind of argument that we have already given in connection with the way in which a given physical system modifies an inertial frame to yield a noninertial one. First, recall that R_f and $R_{f'}$ are respectively determined in f and f' by the same pair of sets ε and τ. When we pass from $f=\Sigma\Sigma(S)/\sim$ to $f'=\Sigma\Sigma(S\cup S_i)/\sim$ the particles of S_i may modify the structure of the points of crossing of S. Assume that this is the case. Then, given that the points of f and f' are equivalence classes of points of crossing with respect to the relation $\sim$ of similarity or of equal structure between points of crossing, the above modification of structure will produce either a redistribution of the modified points of crossing between the points of f to yield points of f', or the appearance of points of f' with a different structure from that of any point of f. This will correspond to a redistribution of the preparticles entering in the points of R_f giving rise to the points of $R_{f'}$. Because of this redistribution of preparticles in passing from R_f to $R_{f'}$ and recalling that in our model the appearance of a point of the trajectory of a physical system S_a in a frame R_f depends uniquely on the condition that preparticles entering in S_a also enter in the point of R_f in question, we can finally understand that although $T(S_a;R_f)$ appears always delocalized in R_f the trajectory $T(S_a;R_{f'})$ may appear localized at least for a given interval of time. Note that in practice when we choose a physical system S_i as a possible detector we do not know this system in detail, we only know certain macroscopic

features of it. Neither do we know in detail the system S producing the field $\Sigma\Sigma(S)/\sim$ in which the frame R_f is defined. Therefore, we do not either know the detailed way in which the field $f_i = \Sigma\Sigma(S_i)/\sim$ will modify the field $f = \Sigma\Sigma(S)/\sim$ to yield the field $f' = \Sigma\Sigma(S \cup S_i)/\sim$ above. Then, in practice, when we choose a S_i as a possible detector it occurs that although we may expect that S_i will serve to detect a physical system S_a we do not know in principle which one will be detected.

Finally, we may say that when a physical system S_a has been localized in a frame $R_{f'}$ by the action of a detector S_i, then S_a manifests itself in $R_{f'}$ as a "particlelike object." If, otherwise, no such localization occurs, then S_a appears in $R_{f'}$ as a "wavelike object".

A more complete treatment of the problem of the detection of particles, or more generally of physical systems, within the framework of our model, will be made elsewhere.

As the last point of this section we will briefly discuss whether uncertainty relations of the same kind as the Heisenberg relations can be stated in our model. Given our definitions of energy, frequency, momentum, and wavelength of a physical system in a frame, the usual argument —according to which the more localized a particle the less well defined its wavelength—applies also to our case. This argument is based on the fact that the wavelength is well defined only for a wave being both regular and of infinite extension in space. In more quantitative terms, assume that for a wave of wavelength λ and having regular oscillations in space within a region of extension $n\lambda$ (n an integer), and vanishing outside this region, we may write $\Delta\lambda/\lambda \sim 1/n$. [For a more complete exposition of this argument see Wichmann (1971, Chap. 6).] Then, from equation (3.20) it follows that $\Delta\lambda/\lambda = \Delta p/p$. Using again equation (3.20) together with $\Delta x \sim n\lambda$, we obtain $\Delta p \Delta x \sim N[R_f] = N_0$. The case in which we maintain that the wave does not vanish only within an extension of $n\lambda$, but omitting the condition concerning the regularity of the wave, will correspond to $\Delta p \Delta x > N_0$. We then may write the relation $\Delta p \Delta x \geqslant N_0$. On the other hand, following similar steps one also arrives at $\Delta E \Delta t \geqslant N_0$. Note that in both these two relations N_0 plays the same role as the Planck constant in Heisenberg uncertainty relations. One may check that although $N_0 \sim 10^{300}$ according to our rough evaluation above, if we express Δp and Δx as numbers of field points in $\Delta p \Delta x \geqslant N_0$, we obtain that the equality $\Delta p \Delta x = N_0$ stands only when Δp and Δx are very small with respect to the number of field points equivalent to the cgs units of momentum and distance. The same occurs in the case of ΔE and Δt in connection with the relation $\Delta E \Delta t \geqslant N_0$.

We have mentioned in Section 1 that in the present paper our assumption stating that the total number of preparticles of the physical world is finite, is in our model related to a general property of physical

systems. This property consists in the fact that energy and frequency are proportional to each other, the constant of proportionality being finite. The finiteness of this constant of proportionality leads immediately to the conclusion that a finite energy corresponds to an equally finite frequency. Yet, this very simple conclusion already considered as a "fact of life" in quantum physics would not be verified in our model, unless we either introduce the assumption of the finiteness of the total number of preparticles, or we consider that the intensity of relevant fields is infinite at every point (this last possibility will considerably complicate our model by the handling of infinite quantities associated to every point of a field). This follows immediately from equation (3.1), which yields that $\rho(x_i; R_f)=0$ for every point x_i of R_f if the intensity $I(x_i; R_f)$ is finite and we assume that $N[B]$ is an infinite number. Then, equations (3.2), (3.7), and (3.10) would imply that to any finite energy corresponds a zero frequency. Therefore, according to this argument we have made the choice of postulating in our model that the total number of preparticles in the physical world is finite.

In this section we have defined energy, momentum, frequency, and wavelength of a physical system in a given frame. We have shown that proportionality relations energy $\propto$ frequency and momentum $\propto$ (wavelength)$^{-1}$ are valid in our model and that both proportionality relations become equalities using the same proportionality constant, which we have interpreted as the Planck constant. Then, the question naturally arises whether according to our definitions of energy and momentum these quantities are conserved in the usual sense. This we will answer in the affirmative in the next section. Also we will show that the relativistic relation between energy, momentum, and rest mass is compatible with our model.

4. ENERGY AND MOMENTUM CONSERVATION

Let us start by asking why a particle or system of particles may be such that the points of their trajectories spread over an arbitrarily large lapse of time in a frame R_f. This is the same as asking in the case of a free particle why this particle will appear in R_f at all times, although the number of the preparticles entering in this particle is finite and even not necessarily very large. The answer to this kind of question is that because one and the same preparticle may belong to more than one field point of R_f (see Section 3) and that preparticles are randomly distributed over the points of R_f, the same preparticles will appear again and again in points of R_f as larger regions of R_f are considered. Since, on the other hand, the condition for a point x of f to belong to a trajectory $T(S_i; R_f)$ is that S and the point $x \in f$ share at least a preparticle, it follows that even if the

number of preparticles entering in S is not very large the points of $T(S_i; R_f)$ may spread over the whole of R_f. The difference between two trajectories $T(S_i; R_f)$ and $T(S_j; R_f)$ such that many more preparticles enter in S_i than in S_j, will reside in the density with which the points of $T(S_i; R_f)$ will appear in R_f with respect to the density corresponding to $T(S_j; R_f)$. In the first case the density will be considerably larger than in the second case, but in both cases the points of the trajectories will spread over the entire R_f. Keeping this in mind, let us analyze the energy and momentum conservation.

Once we specify a system S_i and a reference frame R_f with $f = \Sigma\Sigma(S)/\sim$, the energy of S_i in R_f is determined according to our definition of energy by $N[T(S_i; R_f)]$ [see equation (3.8)]. The usual sense in which energy is conserved can be expressed in our terms as follows. Consider a particular time $t = t_0$ in the frame R_f and call $T_<(S_i; R_f)$ the subset of $T(S_i; R_f)$ such that every point belonging to $T_<(S_i; R_f)$ has a time coordinate $t < t_0$. In the same way, consider the set $T_>(S_i; R_f)$ of points whose time coordinates are equal to or larger than t_0. Assume that R_f is inertial and such that there are very many points in the time axis of R_f corresponding to time coordinates both smaller than t_0 and larger than t_0. Then, according to our definition of inertial frame it follows that we can infer the energy of S_i in R_f by considering only the points of either $T_<(S_i; R_f)$ or $T_>(S_i; R_f)$. For example, this energy can be obtained by multiplying $N[T_<(S_i; R_f)]$ by the ratio $N[R_f]/N[R_{f<}]$, where $N[R_{f<}]$ is the number of points of R_f having time coordinates smaller than t_0. This can be easily understood recalling our definition of inertial frame and that a point x of a trajectory $T(S_i; R_f)$ occurs when a preparticle entering in S_i also enters in the point x of R_f, and that preparticles distribute themselves randomly in the points of R_f. Recall that according to our definition of inertial frame the number of preparticles entering in different regions of R_f covering the same number of field points is the same, apart from fluctuations which become negligible when sufficiently large regions of R_f are considered (see Section 3). Then, the density of points of $T(S_i; R_f)$ in R_f can be considered to all practical effects identical to the density of points of $T_<(S_i; R_f)$ in the region of R_f corresponding to time coordinates smaller than t_0, since this region of R_f is assumed to be very large. Equivalently, the density of points of $T(S_i; R_f)$ in R_f is equal to the density of points $T_>(S_i; R_f)$ in the region of R_f where points have equal or larger time coordinate than t_0. One consequence of this is that the energy $E(S_i; R_f)$ can also be calculated considering only the points of $T_>(S_i; R_f)$ according to the formula

$$E(S_i; R_f) = N[T_>(S_i; R_f)] \frac{N[R_f]}{N[R_{f>}]} \tag{4.1}$$

The fact that we obtain the same energy when considering either of the two partial trajectories $T_<(S_i; R_f)$ or $T_>(S_i; R_f)$ allows us to say that the energy of S_i in R_f is conserved in the usual sense. Another way to express this is that the frequency calculated from $T_<(S_i; R_f)$ is equal to the frequency calculated from $T_>(S_i; R_f)$, as follows directly from our discussion above. Note that what becomes essential to energy conservation here is that the reference frame R_f under consideration be inertial, which, according to our definition of inertial reference frame, can also be interpreted as a homogeneity condition. In turn, this homogeneity condition is fulfilled only if the intervals of time both before $t = t_0$ and after $t = t_0$ that we consider are sufficiently large, since in the opposite case differences between the number of preparticles involved in different regions of R_f covering the same number of field points, but occurring respectively before and after $t = t_0$ in R_f, may be important. This is equivalent to saying that energy conservation can be violated for sufficiently short intervals of time.

The above discussion applies also in the case in which we are concerned with more than one physical system S_i. Let S_i and S_j be two physical systems and consider the energy $E(S; R_f)$ of the system $S = S_i \cup S_j$. Assume that the trajectories $T(S_i; R_f)$ and $T(S_j; R_f)$ appear quite separate in R_f except in a small region of R_f whose points have time coordinates closely around $t = t_0$, and where these two trajectories cross each other. Then, our argument above applies again to this case and leads us to say that the energy of the global system $S = S_i \cup S_j$ is conserved in the "process" of intersection between trajectories $T(S_i; R_f)$ and $T(S_j; R_f)$. More generally, the same applies when one is concerned with an arbitrary number of physical systems.

The argument in favor of the momentum conservation in our model is more intricate than the one we have given above for the energy conservation. Let us analyze this problem for the case of an inertial frame R_f such that the space–time diagram associated to it has only two space axes, which we label x and y. Then consider the collections C_x and C_y of all the systems which have in R_f a momentum pointing along the positive directions of the x and y axes, respectively. Let us now denote as $\pi(S)$ the set of all the preparticles entering in S. Consider a physical system S_a and select from C_x the set S_{xa} the most similar to S_a in what concerns the preparticles entering in S_{xa} and S_a. Then let us define the set $\pi(S_{xa}) \cup \pi(S_a) - \pi(S_{xa}) \cap \pi(S_a) \equiv \pi_{xa}$ such that the number of preparticles belonging to π_{xa} measures the extent to which the sets $\pi(S_{xa})$ and $\pi(S_a)$ are different from each other. In the same way, we can choose from C_y the set S_{ya} the most similar to S_a and define a set π_{ya}. According to the definition of C_x, C_y, S_{xa}, S_{ya}, π_{xa}, and π_{ya} it follows that if $\pi_{xa} = \phi$ the momentum $\mathbf{p}(S_a; R_f)$ points along the positive x axis, if $\pi_{ya} = \phi$ then $\mathbf{p}(S_a; R_f)$ points along the positive y axis, and if $\pi_{xa} \neq \phi$ and $\pi_y \neq \phi$ then $\mathbf{p}(S_a; R_f)$ points in a different direction than $\mathbf{x}$

and y. Now, consider here again as in our discussion on the conservation of energy an instant $t = t_0$ and the two regions R_f whose points have time coordinates respectively smaller than t_0 and equal or larger than t_0. Denote these regions $R_{f<}$ and $R_{f>}$. The sets $T_<(S_a; R_f)$ and $T_>(S_a; R_f)$ will be subsets of the trajectory $T(S_a; R_f)$ having as elements the points of $T(S_a; R_f)$ with time coordinates either smaller or larger than t_0. Then $\mathbf{v}(S_a; R_{f<})$ and $\mathbf{v}(S_a; R_{f>})$ will be velocities corresponding respectively to the points of the subsets $T_<(S_a; R_f)$ and $T_>(S_a; R_f)$ of the trajectory $T(S_a; R_f)$, to which correspond the velocity $\mathbf{v}(S_a; R_f)$. We can now ask how long must a lapse of time in R_f be in order that differences in direction and magnitude between $\mathbf{v}(S_{xa}; R_f)$ and $\mathbf{v}(S_a; R_f)$ due to $\pi_{xa} \neq \phi$, and between $\mathbf{v}(S_{ya}; R_f)$ and $\mathbf{v}(S_a; R_f)$ due to $\pi_{ya} \neq \phi$ may clearly manifest themselves. The answer is that since preparticles distribute themselves randomly in the points of R_f, and that in such points enter the same number of preparticles when this number is averaged over a large region of R_f, then by considering a sufficiently long lapse of time the relations between the vectors $\mathbf{v}(S_{xa}; R_f)$, $\mathbf{v}(S_{ya}; R_f)$ and $\mathbf{v}(S_a; R_f)$ appear clearly as if we had considered all the time axis of R_f, i.e., all the point of R_f. As a consequence, the momentum calculated according to any of the expressions $\mathbf{p}(S_a; R_{f<}) = E(S_a; R_{f<})$ $\mathbf{v}(S_a; R_{f<})$, $\mathbf{p}(S_a; R_{f>}) = E(S_a; R_{f>})$ $\mathbf{v}(S_a; R_{f>})$ or $\mathbf{p}(S_a; R_f) = E(S_a; R_f)$ $\mathbf{v}(S_a; R_f)$, has the same value provided that sufficiently long lapses of time are considered both before and after $t = t_0$.

Following a similar argument as above we can also say that the magnitude of $\mathbf{p}(S_a; R_f)$ is a measure of the extent to which the preparticles entering in S_a are different from those entering in a system S_{0a} selected as the most similar to S_a from the collection C_0 of all the systems appearing at rest in R_f. Note the contrast with the energy $E(S_a; R_f)$ which appears to be related with the number of preparticles entering in S_a and the average number of preparticles entering in one point of R_f.

As the last point of this section let us describe how in the present model it is possible to see the change of the energy of a system S_i in an inertial frame R_f due to the interaction of S_i with another system S_j. To this end, consider the trajectories $T(S_i; R_f)$, $T(S_j; R_f)$ and $T(S; R_f)$, where $S = S_i \cup S_j$. Assume, in addition, that $T(S_i; R_f)$ and $T(S_j; R_f)$ cross each other in the region of R_f around the point with coordinates (x_0, t_0). When such crossings occur the question arises as to whether the representation of $T(S; R_f)$ in the space–time diagram associated to R_f is so different from the simple superposition of the respective representation of $T(S_i; R_f)$ and $T(S_j; R_f)$ that $T(S; R_f)$ cannot be described even roughly in terms of $T(S_i; R_f)$ and $T(S_j; R_f)$. As an example of such crossings we can think of the collision occurring in R_f between two physical systems respectively represented by S_i and S_j. If the collision does not involve too much energy

then we are concerned with the same particles or systems during the whole process and the changes occurring are ascribed to variations in the energies and momentum of these particles or systems. As is well known, when exchanges of energy are either comparable or larger than the rest energy of the colliding particles, then we can no longer say that we are concerned with the same particles in the whole process. Since we are discussing the change of energy of a physical system due to interactions with the systems, we are more concerned with the first case. Thus, we assume that the trajectory $T(S; R_f)$ is such that before $t = t_0$ the points of this trajectory can be separated in two subsets $TS_{i<}$ and $TS_{j<}$ and after $t = t_0$ in the two subsets $TS_{i>}$ and $TS_{j>}$; also that $TS_{i<} \cup TS_{i>}$ and $TS_{j<} \cup TS_{j>}$ can be unambiguously ascribed to S_i and S_j, once the interactions between these systems have been taken into account. One criterion to decide what we ascribe to S_i and what to S_j in the trajectory $T(S; R_f)$ could be the way in which the points of this trajectory distribute themselves in R_f. In the present paper we have not faced the problem of which are the probability distributions appropriate to describe different cases of trajectories $T(S; R_f)$. If nonuniform distributions are allowed then one criterion to follow is to consider regions of R_f surrounding loci of maximum density of points of $T(S; R_f)$. Assume in our example above that we represent in the space–time diagram of R_f only those points of trajectories falling in regions around in which the density of points of $T(S_i; R_f)$, $T(S_j; R_f)$, and $T(S; R_f)$ present maxima. Then one possibility is that $T(S_i; R_f)$ and $T(S_j; R_f)$ may each appear as a stripe of points in the space–time diagram associated to R_f, and $T(S; R_f)$ as two stripes of points crossing each other in R_f and respectively similar to those corresponding to $T(S_i; R_f)$ and $T(S_j; R_f)$.

Let us consider that the set of points $TS_{i<}$ above is such that the velocity corresponding to it is zero in R_f. If the velocity corresponding to $TS_{i>}$ is equal to $v \neq 0$ we say that before $t = t_0$ the system S_i was at rest in R_f and that after the crossing with S_j which occurs at $t = t_0$ the system S_i acquires the velocity v. In the same way, the energy of S_i before $t = t_0$ will be $N[TS_{i<}]\ N[R_f]/N[R_{f<}] \equiv N_M$, and $N[S_{i>}]\ N[R_f]/N[R_{f>}] \equiv N_E$ after $t = t_0$, where $N[R_{f<}]$ and $N[R_{f>}]$ are the number of points of the regions $R_{f<}$ and $R_{f>}$ corresponding to all those points of R_f with $t < t_0$ and $t > t_0$, respectively. A simple mean-value calculation, which does not take into account the shape of the appropriate probabilistic distribution of the points of trajectories in R_f, but only the corresponding mean values, can be the following one:

$$N_M O + N_{\Delta E} \alpha = N_E v \tag{4.2}$$

where N_E is the energy of the resulting system after the interaction has

taken place. The number $N_{\Delta E} \equiv N_E - N_M$ is the number of points of the trajectory of the resulting system after $t = t_0$ which are in excess with respect to the numbers of points of the trajectory of the original system at rest in R_f. In this oversimplified view of energy transfer to a physical system it is assumed that all the points of the trajectory of the system originally at rest in R_f plus some or all the points of the trajectory of another system [there are $N_{\Delta E}$ of such points in equation (4.2)] belong to the trajectory of the resulting system; the number $N_{\Delta E}$ being the energy transfer to the system originally at rest in R_f. Then α in equation (4.2) stands for the mean contribution to the velocity v of the resulting system coming from those points of its trajectory arising due to the energy transfer. A first guess could be $\alpha = 1$, which is equivalent to saying that the subset of the trajectory of a system corresponding to the energy transfer to another system behaves as light in the sense that the points corresponding to the energy transfer are randomly distributed. The assumption $\alpha = 1$ together with equation (4.2) lead immediately to $N_{\Delta E} = N_E v$ which is equivalent to saying that $N_{\Delta E}$ is the magnitude of the momentum of the resulting system after $t = t_0$, since N_E is the energy of such a system and we have the relation $\mathbf{p} = E\mathbf{v}$, according to the definition of momentum (see Section 3). On the other hand, following the oversimplified model of energy transfer here analyzed we also have

$$N_M + N_{\Delta E} = N_E \tag{4.3}$$

which together with equation (4.2) and $\alpha = 1$ yield $N_E = N_M/(1-v)$ and $N_{\Delta E} = N_M v/(1-v)$. Since, on the other hand, for the case in which the resulting physical system has a velocity along the negative direction of the x axis of R_f such that equation (4.2) becomes $N_M O + N_{\Delta E}(-\alpha) = N_E(-v)$, then we can write $E = M/(1-|v|)$ and $\mathbf{p} = Mv/(1-|v|)$, where $M = N_M$ and $E = N_E$. Note that these two relations for energy and momentum lead to $M + |p| = E$ and that this linear relation between E, $|p|$, and M is not acceptable with relativity. As is well known, this condition of compatibility is fulfilled by $M^2 + p^2 = E^2$. However, the above expressions for E and p present the features of special relativity of yielding $E = M$ and $p = 0$ when $v = 0$ and $E = p = \infty$ when $|v| = 1$. On the other hand, $\mathbf{p} = M\mathbf{v}/(1-|v|)$ reduces itself to the Newtonian expression $\mathbf{p} = M\mathbf{v}$ in the case that $|v| \ll 1$, but unfortunately, the power series development of $E = M/(1-|v|)$ contains a linear term in $|v|$ and therefore it does not give the Newtonian expression for the energy when $|v| \ll 1$. The serious defects presented by the above expressions for energy and momentum are not surprising since they result from very crude assumptions on the process of energy transfer as we have mentioned above.

One way to modify the above crude model of energy transfer in order to obtain a relation between E, $\mathbf{p}$, and M compatible with relativity consists in loosening the restriction $\alpha = 1$ in equation (4.2). In this sense, having as a requisite that the mean velocity contribution coming from those points of the trajectory corresponding to the energy transfer must be such that $\alpha \leqslant 1$, it can be seen that the relation

$$\alpha = (1/v)(1 - v^2)^{1/2} \tag{4.4}$$

leads to

$$E = \frac{M}{(1 - v^2)^{1/2}} \tag{4.5}$$

Also, according to the definition of momentum, we obtain $\mathbf{p} = E\mathbf{v} = M\mathbf{v}/(1 - v^2)^{1/2}$. These expressions for energy and momentum are the relativistic ones and they lead immediately to the relation $E^2 = M^2 + p^2$. Note that from equations (4.2) and (4.4), and the definition of momentum $\mathbf{p} = E\mathbf{v}$, it follows that now $N_{\Delta E} \neq |p|$ and that equation (4.2) still holds true together with $E^2 = M^2 + p^2$.

It is clear that to assume equation (4.4) is to force things in order that a very rough model of energy transfer be compatible with special relativity. In this sense, equation (4.4) can be interpreted as a relation between α and v necessary to correct the too crude assumption according to which the trajectory of the resulting system—once the energy transfer has been taken into account—comprises both the points of the trajectory of the original system and the points corresponding to the energy transfer $N_{\Delta E}$. Something less crude could be elaborated if one takes into account the shape of the probabilistic distributions of the points of trajectories of the physical systems having velocities smaller than unity in the reference frame under consideration. Then the problem of transfer of energy between two systems S_i and S_j could be seen as the modification of the probabilistic distribution of points belonging to $T(S_i; R_f)$ due to the interaction between S_i and S_j. The effects of this interaction would be reflected in the probabilistic distribution of the points of $T(S; R_f)$, where $S = S_i \cup S_j$. In particular, the regions of R_f where such a probabilistic distribution presents maxima could be shifted with respect to the regions of R_f where the maxima of the probabilistic distributions corresponding to $T(S_i; R_f)$ and $T(S_j; R_f)$ appear. Such modifications of the position of the maxima may correspond to a change in velocity if we assume that particles or systems of particles are found around the regions where the probabilistic distribution of the points of their trajectories present maxima values. For instance, assume that the

regions around the maxima of $T(S_i; R_f)$ and $T(S_j; R_f)$ can be represented by two stripes of points crossing each other in R_f and each having a well-defined slope with respect to the t axis, then the regions of R_f around the maxima of $T(S_i \cup S_j; R_f)$ could appear in R_f as a four-branch pattern. Following a criterion of smooth linking of consecutive branches one could ascribe two consecutive branches to S_i and the remaining branches to S_j. If the inclination with respect to the t axis in R_f changes when one passes from one branch before the crossing to the branch associated to the same system after the crossing, then this corresponds to a change in velocity of the system in question. Note that the displacement of the region around the maxima of probabilistic distribution occurs not only because we are taking into account the shape of probabilistic distributions and then by simple addition of these distributions such displacements may be expected; but also because these displacements of the maxima are influenced by the interaction between S_i and S_j. Such interaction could make the introduction of a correction to the simple addition of the probability distributions corresponding to $T(S_i; R_f)$ and $T(S_j; R_f)$ necessary in order to get the probability distribution corresponding to $T(S_i \cup S_j; R_f)$. In the search of the appropriate general probabilistic distributions of the points of the trajectory of physical systems in a given frame, one could use the guiding requirement that the mentioned probabilistic distributions be such that when we pass from the two distributions corresponding respectively to two given systems S_i and S_j, to the distribution corresponding to the global system $S_i \cup S_j$, then energy and momentum transform themselves according to relativity.

Another way to look at this question consists in considering first the space and time coordinate transformations. According to our model, the points of a reference frame are field points. We can limit ourselves to coordinate transformations between frames covering the same region of a global field (space–time), i.e., such that the same field points enter in both. What changes when we pass from one frame to another is the way in which space and time coordinates are assigned to field points in each of these frames. This results from the fact that when we pass from such frames to others the corresponding ϵ and τ sets change and thus the way in which the points are connected to each other in each of these frames. The concept of field point is then playing here the same role as the concept of event in the usual formulation of special relativity. According to this formulation, the collection of events is always the same whatever the reference frame under consideration may be and regardless of changes in the way in which these events appear when one changes reference frames. One is always concerned with the same events. Only the coordinates ascribed to each of these events change in passing from one frame to another.

In Section 3 we have shown that considering the velocity of light as an average value this velocity remains invariant when one passes from one inertial frame to another. As is well known, the Lorentz transformations are linear and continuous transformations that reduce themselves to Galilean transformations when $v \ll 1$, and that are compatible with the invariance of the velocity of light. Since such invariance appears in our model (provided we consider the velocity of light as an average value), the Lorentz transformations appear here again as the appropriate ones, at least for the cases in which we are concerned with reference frames and trajectories both having a very large number of field points. (Other coordinate transformations compatible with the scheme illustrated in Figure 2, for instance those corresponding to $\theta \neq \phi$ in Figure 2, will be studied elsewhere.) On the other hand, we have shown in Section 3 that energy and momentum are conserved in our model if regions of a frame covering a sufficiently large number of points are considered, which is again a condition of macroscopicity. Therefore, if this kind of requirement is satisfied, we can follow the usual procedure leading to the relativistic expressions for energy and momentum as those compatible with Lorentz transformations and conservation of energy and momentum, and as those reducing themselves to the Newtonian expressions when $v \ll 1$.

Finally, let us analyze in the framework of our model how it may occur that a system S_i which has acquired energy in an interaction with another system with respect to a given frame R_f is such that there exists another reference frame with respect to which the energy of S_i is equal to the energy of this system in R_f, before the interaction has taken place. This is what occurs for instance in an elastic collision. Let N_M be the energy of the system S_i under consideration at rest before the collision with another system occurring at $t = t_0$ in the inertial frame R_f. Then N_M is also the mass of S_i. Let N_E be the energy of S_i after the collision ($t > t_0$) and v the velocity acquired by S_i in the frame R_f as a result of this interaction. Therefore we can write $N_E = \lambda N_M/(1-v^2)^{1/2}$, where $\lambda = 1$ or $\lambda > 1$ depending on whether the collision is elastic or not. On the other hand, let N_u be the unit of mass in terms of which N_M is expressed. (In particular, N_u may be equal to 1, which is the unit we have chosen in all our discussions above.) Then the energy of S_i in R_f respectively before and after the collision has taken place will be N_M/N_u and N_E/N_u. Consider now a reference frame $\tilde{R}_f$ moving with respect to R_f with the same velocity as the system S_i at times $t > t_0$. Then, the system S_i appears at rest in $\tilde{R}_f$ for all times $t > t_0$. Let S_u be a physical system serving as a standard to measure the energy of physical systems at rest in R_f. Thus S_u is at rest in R_f and N_u is the number of field points belonging to the trajectory $T(S_u; R_f)$. If we want to measure the energy N_E of S_i in the frame $\tilde{R}_f$ where S_i appears at rest for $t > t_0$, we must use the standard S_u, but once this standard has

acquired the velocity necessary to appear at rest in $\tilde{R}_f$, i.e., at rest with respect to S_i for $t > t_0$. Then the energy of the standard will now be $N'_u = N_u/(1-v^2)^{1/2}$, assuming that one tries to perturb as little as possible the standards and therefore one requires that the interaction or collision transmitting the necessary energy to S_u in order to move it at the same velocity as $\tilde{R}_f$ be an elastic process. Thus we can write $N_E/N'_u = \lambda N_M/N_u$, where $\lambda = 1$ and $\lambda > 1$ correspond, respectively, to an elastic and nonelastic collision of S_i. If $\lambda = 1$ we are in the case in which S_i has in R_f for $t < t_0$ the same energy as S_i in $\tilde{R}_f$ for $t > t_0$. On the other hand, it is clear that if $\lambda > 1$ then $N_E = \lambda N_M N'_u/N_u = \lambda N_M/(1-v^2)^{1/2}$, where λN_M is precisely the mass of S_i in $\tilde{R}_f$ for all times $t > t_0$ in the case that S_i has experienced a nonelastic collision at $t = t_0$.

5. CONCLUDING REMARKS

In the present paper we have explored the possibility of formulating a foundation theory of space–time and its relatives in the framework of a simple model. This theory presents the following features.

(i) It is a relational theory in the sense that the disappearance of the most basic ingredients of the physical world (preparticles) would likewise entail the disappearance of physical systems and space–time. However, a space–time is not in our model a network of relations between physical objects, but a physical entity constructed with the same raw materials (preparticles) as any physical system. On the other hand, space–time has in our model a more elaborated mathematical structure than that of physical systems. In this sense, the structure that physical systems may present is in part due to the fact that we describe physical systems in reference frames, which in turn are defined within a given space–time.

(ii) We have defined space–time always with respect to a given collection of physical systems, those of interest in the problem at hand. This flexibility is a convenient feature since though one always speaks of space–time as a unique entity, one frequently refers to "regions" or "parts" of space–time. Furthermore, this feature of our model also allows us to include the case of space–times in which all kind of cuts and nonuniformities occur since these accidents will or will not be present depending on the physical systems with respect to which we define the corresponding space–time.

(iii) Our model can be considered as a pregeometry in the following sense. We have started from only two primitive concepts, those of preparticle and membership relation of set theory, neither of these two concepts being related to the concept of distance. Neither does this last concept

appear in any of the assumptions or the central postulate Postulate 1 we have introduced in our model. Yet, as an elaboration starting from the above two primitive concepts we have obtained as derivative concepts those of distance or separation in space and of interval of time between any two points of a reference frame. Furthermore, we can ascribe a topology to every field (García-Sucre, 1978a). For arguments in favor of the elaboration of a pregeometry, see Misner et al. (1970).

(iv) The postulate according to which two points of space–time are distinct from each other only because they have different structures —independently of whether they share none, some or all preparticles—plays an important role in our model. For instance, in virtue of this postulate we do not need to introduce any extra labels (as numbers, letters, etc.) in order to characterize the point of a space–time with respect to any other point of the same space–time. Another feature illustrating the role that Postulate 1 plays in our model is the property according to which the same physical system may appear either delocalized or localized in a given reference frame, depending on the detector physical system we use in the detection process. Note that we have also stated in terms of the two primitive concepts of our model the concept of detector physical system and what a detection process consists of.

(v) We have seen how our assumption according to which the number of preparticles in the universe is finite is related to the fact that with simple definitions of energy, momentum, frequency, and wavelength of a physical system in a reference frame, we obtain the well-known relations $E = A\nu$ and $p = A\lambda^{-1}$, where A is a constant of proportionality, which we interpret as the Planck constant. Then, from our point of view, these two relations together with the fact that the measured value of the Planck constant is a finite quantity may be taken as an indication that the number of basic ingredients of the universe is also finite.

(vi) Energy and momentum are conserved provided we consider sufficiently large regions of the space–time diagram associated to the reference frame under consideration. Also we have shown that the Lorentz transformations are the appropriate ones in our model provided we introduce some of the usual restrictions. The same occurs with the relativistic expressions for energy and momentum. We have also examined this last question in the framework of a very crude model of energy transfer between physical systems. We have shown that introducing a further restriction to this model we obtain the relativistic expressions for energy and momentum. Yet, we have thus only proved that our model is compatible with special relativity. On the other hand, we have outlined as a possible further development a way in which the relativistic expressions for energy and momentum may be obtained in our model as a consequence of

the form of the general probabilistic distributions describing the way in which the points of trajectories of physical systems having velocities $v < 1$ distribute themselves in reference frames.

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